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Hedging mortgage risk with T-bond futures and options: A case study

by

Daniel Curtis Messerschmidt

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CHAPTER I. INTRODUCTION

Statement of the Problem

Financial institutions of many types historically have financed holdings of long-term maturity assets by borrowing for shorter terms. During periods when interest rates were stable and the yield curve maintained an upward slope, this strategy resulted in profits from the spread in rates. Starting in the mid-1960s, significant upward shifts in the yield curve resulted in losses for many institutions which pursued the strategy of borrowing short and lending long. This was especially true for savings and loan associations which have traditionally held the major part of their asset portfolios in fixed interest rate mortgages with original maturities of twenty years or more, and which have financed these asset positions by accepting short-term savings and time deposits. The pronounced upward movements in interest rates that have been especially prevalent since October 1979 have created periods of substantial losses for individual savings and loan associations, and, during some periods, significant losses for the industry as a whole.

The recent increase in volatility of interest rates in the United States has spawned a rapidly growing interest among both academicians and practitioners in the financial industry in hedging against the risk of upward shifts of the yield curve. The asset/liability maturity gap traditionally maintained by savings and loan associations has generated special interest and greater urgency among these institutions in developing strategies to reduce interest rate risk. The introduction of trading in Government National Mortgage Association Collateralized Depository

Receipts futures contracts in October, 1975, and in Treasury bond futures contracts in August, 1977, provided two alternative instruments which could be used to hedge against adverse interest rate movements.[1]

An increase in long-term interest rates will cause the value of mortgage portfolios to decline; the longer the term to maturity of these assets, the greater will be the effect on their market value. To the extent that a rise in interest rates causes the prices of mortgages and interest rate futures to move together, a holder of mortgages can hedge against interest rate movements by selling futures contracts. However, to the extent that mortgage and futures move together, a strategy of selling futures contracts to offset adverse movements in the cash value of a portfolio also eliminates the possibility of gains when long-term interest rates decrease and the market value of asset holdings rises. Thus, a strategy of hedging against interest rate risks using futures instruments will reduce the possibility of both downside loss and upside gain if cash and futures instruments' prices move together.

The volatility of interest rates in recent years and the inception of trading in interest rate futures contracts has spawned a substantial body of literature concerned with both the theoretical foundations of hedging fixed interest rate asset and/or liability positions, and empirical tests of the efficacy of alternative hedging strategies. The basis for much of this research is the pioneering article of Ederington (1979), which combined portfolio and futures market theory to derive the risk

[1]These contracts are traded on the Chicago Board of Trade. The description of the structure of these contracts and the history of the development and market size of trading are presented in Chapter Two.

minimizing hedge ratio. Since the publication of Ederington's work, a number of articles have been written on the effectiveness of interest rate futures instruments for hedging holdings of fixed-rate assets.

Few of these studies have focused on hedging mortgage portfolio holdings using interest rate futures. Hyman and Jaffee (1981), Kolb, Corgel and Chiang (1982) and Starleaf and Langley (1983) have analyzed the risk reduction potential of alternative mortgage hedging strategies. However, only Starleaf and Langley have tested these strategies empirically.

With the inception of trading of options on Treasury bond futures contracts in October, 1982, an additional instrument which could be used for hedging against interest rate risk became available. An option on a futures contract gives the buyer the right to purchase (a call option) or sell (a put option) the futures contract. The advantage of options over the actual purchase or sale of futures is that the buyer of the option has the right, but not the obligation, to exercise the option. In periods when the prices of mortgages and interest rate futures fall, the holder of a put option could exercise his/her right to sell the futures instrument at a higher price than the current price of the futures contract, thus offsetting the loss on the asset holding with a gain on the futures contract. In periods when mortgage and futures prices rise, the holder of a put option would not be required to exercise his/her option. The result would be that the gain on the asset holding would not be offset by a loss on the futures contract.[2]

[2]Options on Treasury bond futures are currently traded on the Chicago Board of Trade. Contract specifications and market characteristics are presented in Chapter Two.

The literature on the use of futures options in hedging strategies has focused on the theoretical foundations of options strategies or the presentation of examples of strategies which could be used to hedge against adverse interest rate movements. To date, only Hyman and Jaffee have analyzed the use of options in mortgage portfolio hedges, but their paper contained no empirical testing of these strategies.[3]

The purpose of the present study is to develop alternative hedging strategies that could be used to hedge mortgage portfolios against interest rate risk through the use of combinations of futures and/or options contracts, and to evaluate empirically the risk-return distributions of the alternative strategies. The structure of the Treasury bond futures contract and the options contracts on this instrument are presented in Chapter II. Additionally, the development and size of the market are discussed.

The theoretical foundations of hedging and options pricing models are developed in Chapter III. The major empirical tests of the hedging and options pricing models are also discussed in this chapter. The results of the empirical tests of the option pricing model used in this study are presented in Chapter IV.

The alternative hedging strategies used are developed in Chapter V, as well as the empirical testing of the risk-return distributions of

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[3]The Chicago Board of Trade has published a number of reference guides which provide examples of options strategies using Treasury bond futures options. Two recent publications are *Strategies for Buying and Writing Options on U.S. Treasury Futures* (1982b) and *Options on Treasury Bond Futures for Institutional Investors* (1983c). Articles in financial publications which analyze options strategies for hedging mortgage portfolio risk include Abbott (1983), Morrissey (1983) and Hartzog (1983).

them. The primary areas presented are the data description, delineation of alternative strategies, estimation of the optimal hedge ratio, and statistical estimation of the risk-return distributions of the strategies.

The summary of the results, limitations of the study, and suggestions for further research are presented in Chapter VI.

CHAPTER II. INSTITUTIONAL CONSIDERATIONS

To understand methods of hedging using Treasury bond futures and options on Treasury bond futures, it is essential that one be familiar with the most important institutional details of these markets. Therefore, a description of the Treasury bond futures and futures options markets is presented in this chapter.

Treasury Bond Futures: Institutional Features

A Treasury bond futures contract is a firm commitment by two parties to make or take delivery of one or more deliverable grade U.S. Treasury bonds during a specified month in the future at a specified price. The contract has standardized terms and conditions established by the Chicago Board of Trade, and the contracts are traded on this exchange. The seller of a contract is said to take a short position, and the purchaser of the contract takes a long position. The basic trading unit of the contract is $100,000 par value of U.S. Treasury bonds with eight percent coupons. Deliverable grade bonds are those that mature at least fifteen years from the delivery date if not callable, and if callable are not so for at least fifteen years from the delivery date.

The price at which a specific contract is traded is established on the trading floor of the Board of Trade. Prices are quoted as a percentage of the par value of the contract and have a minimum fluctuation of 1/32 of a point, which is $31.25 per contract. A price quotation of 74-03 represents a price of 74.09375 percent of par, or $74,093.75 for one contract. Daily price limits establish the maximum movement of the price of a contract for any one trading day and are set at 64/32, or $2,000, above and below the previous day's settlement price. Variable limits, currently $3,000, establish a larger range for price movements when the price of the future has closed up or down the limit for three consecutive days in three or more contract months during a business year. The settlement price is the average price for all trades completed during the last two minutes of the trading day.

Currently, contracts are traded for four delivery months during each year -- March, June, September and December. At any time there is trading in contracts that mature in more than one year. For example, on June 20, 1984, a total of eleven contracts were traded: three for 1984 maturity dates, four for 1985, and four for 1986. Thus, a trader could buy or sell a contract which matured approximately two and one-half years hence.

Each contract expires during the month of the contract date with no trades taking place during the last seven days of the business month. If a position is not offset prior to expiration of the contract, the holder of the short position (the seller of the contract) decides which eligible Treasury bonds to deliver. If the bonds to be delivered do not have a coupon rate of interest of eight percent, the price at which they are invoiced is determined such that the yield of the bonds is eight percent. The Board of Trade determines conversion factors for all eligible bonds for each futures contract expiration date. These conversion factors are based on the coupon rate of the Treasury bond and its time to maturity or, if callable, the first call date. There were twenty-four deliverable grade Treasury bonds for the June, 1984, futures contract. A sample of these is presented in Table 2-1, with different coupon rates and maturity or call dates.[1]

Table 2-1. T-bonds deliverable on the June, 1984, T-bond futures contract

Coupon	Maturity	Conversion factor
7 5/8	February 15, 2002-07	.9650
8 3/4	November 15, 2003-08	1.0728
10	May 15, 2005-10	1.2007
13 1/8	May 15, 2001	1.4681
15 3/4	November 15, 2001	1.7180
11 5/8	November 15, 2002	1.3446
10 3/4	May 15, 2003	1.2645
11 5/8	August 15, 2003	1.3026

The seller of the futures contract delivering an eligible bond calculates the invoice amount for a particular bond by multiplying the futures settlement price by the appropriate conversion factor and adding any accrued interest. The buyer of the contract on which delivery is made then takes on an investment with a yield equal to the yield he/she would have received had he/she paid the settlement price and received an eight percent coupon. Assume that a June, 1984, T-bond futures contract has a settlement price of 82-20. If the seller chooses to deliver the May 15, 2005-10 contract, the conversion factor is 1.2007. The invoice price is:

$$\text{Invoice Amount} = .82625 \times 1.2007 \times \$100{,}000 = \$99{,}207.84.$$

Thus, the seller would invoice the buyer $99,207.84 plus any accrued

[1]The data on Treasury bond futures trading for June 20, 1984, is presented in Table 2.2 and is taken from the *Wall Street Journal* of June 21, 1984.

interest on the bonds.[2] Although it is important to understand the delivery process, a large percentage of T-bond futures positions are offset before expiration. This is done by a seller (purchaser) of a contract purchasing (selling) another contract for the same maturity date. As an example of the relatively infrequent delivery, in 1980, less than one-half of one percent of all T-bond futures contracts were settled by delivery.

There are two types of costs of trading in futures contracts -- margin and commission costs. Each purchaser or seller of a contract must post an initial margin; currently this is $2,000 per contract.[3] The margin is posted with the Chicago Board of Trade Clearing Corporation and is held until the trader offsets his/her position or the contract expires. The required margin for a hedged position is currently $1,500. This initial margin is a performance bond which ensures that the trader makes his/her potential losses good in advance. There is also a maintenance margin which is the additional deposit a trader must make to hold a deposit when the price of the contract moves adversely for his/her position. For example, if the maintenance margin is $2,000 and Treasury bond futures prices fall by 40/32 of a point, the purchaser of a T-bond futures contract would lose $1,250 and must, therefore, deposit sufficient funds to

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[2]The invoice amount is calculated as:

Settlement price in decimal equivalents x conversion factor x contract size + accrued interest.

In this instance, the invoice amount is:

Invoice amount = .82625 x 1.2007 x $100,000 + accrued interest
= $99,207.84 + accrued interest.

[3]These margins are established by the Board of Trade, are subject to change, and may not reflect actual margin deposits required by member firms.

bring his/her account back to the initial margin. However, any gains on price fluctuations can be immediately withdrawn. In this example, the seller of the contract could withdraw his/her $1,250 if the initial margin deposit were maintained. These positions are "marked-to-market," which means that they are settled after each trading day, and a trader for whom the price movement is adverse must add the required funds to his/her account by the start of the next trading day. The cost of the margin account to a trader is then the interest earnings foregone on the margin deposit.

Commission costs are those charges assessed by clearing members for undertaking trades. These fees vary among firms, the size of the trade, and the customer's relationship to the trading firm, but are generally in the neighborhood of $50. Commission charges are assessed on a "round turn" basis, i.e., the commission fee covers both the entry into and exit from the futures market.

The Size of the Treasury Bond Futures Market

Since the inception of trading in Treasury bond futures on July 22, 1977, this contract has become the most actively traded of any futures contract in the world. The volume of trading and number of outstanding contracts is the open interest. Open interest figures are for the total number of contracts outstanding, not the sum of positions taken. As of June 19, 1984, there were 195,684 contracts outstanding for all contracts traded on this date (see Table 2-2). Generally, trading is largest for contracts closest to maturity, as is indicated by the 138,261 outstanding September, 1984, contracts which was 70.7% of all outstanding contracts.

Although there were only 22,889 outstanding contracts for the June, 1984, contract, this is not unusual, because it was close to the contract expiration date and most positions had already been offset.

Table 2-2. Treasury bond futures data for June 20, 1984

Contract	Settlement price	Open interest
June 1984	61-19	22,889
September 1984	60-23	138,261
December 1984	60-06	15,519
March 1985	59-25	6,256
June 1985	59-14	4,181
September 1985	59-06	1,215
December 1985	58-31	1,419
March 1986	58-25	1,845
June 1986	58-20	3,133
September 1986	58-16	884
December 1986	58-13	82
	Total open interest	195,684

Even though the market for contracts farther from maturity is thinner than for nearby contracts, there is still substantial trading in those contracts with maturities from six to twelve months in the future -- those contracts which could be used for hedges of this length. There is also substantial trading each day. On June 20, 1984, the total volume of contracts traded was 127,107, almost two-thirds of the outstanding contracts. There has been a steady growth in volume since the inception of trading. In 1978, average daily volume was only approximately 2,000 contracts; by 1980, this volume had grown to approximately 33,000 contracts and, in 1982, it was approximately 61,000 contracts.

Options on Treasury Bond Futures:
Institutional Features

Trading in options on Treasury bond futures was initiated in October, 1982. There are two types of options, puts and calls, and a trader can either write (sell) or purchase an option. A purchaser of a call (put) option on a Treasury bond future acquires the right to assume a long (short) position in one T-bond futures contract of a specified month at a price (the strike or exercise price) established when the option is purchased. The seller, or writer, of a call (put) option has the obligation of taking a short (long) position in the futures market if the buyer of the option exercises his/her right. A writer of a call (put) option agrees to sell (buy) one T-bond futures contract at a specified price if the buyer decides to exercise the option. T-bond futures options are American options, which means that the holder of the option has the right to exercise it at any time prior to its expiration. Options which can be exercised only at expiration are called European options.

The trading unit for options contracts is one $100,000 face value Chicago Board of Trade U.S. Treasury bond futures contract of a specified contract month. Trading months for options are the same as for T-bond futures: currently March, June, September, and December. The price at which the futures contract can be purchased or sold is called the strike or exercise price and trading is conducted at prices in integral multiples of two points ($2,000) per T-bond futures contract. On June 20, 1984, exercise prices for call options on the September, 1984, futures contract ranged from 58 to 80 in increments of two points (see Table 2-3). A strike price of 64 on a call option would mean that the purchaser of the call could purchase one $100,000 par value T-bond futures contract for $64,000, and, if the option is exercised, the writer of the option would be obligated to take a short position (sell the contract) at this price. These strike prices are established at the initiation of trading of the options contract; when the price of the underlying futures contract is midway between the highest two, or lowest two, existing strike prices, a new strike price is added for the contract.

The price paid for the option is called the option premium and is determined by open outcry on the trading floor of the Board of Trade. These prices are quoted in multiples of 1/64 of one percent of a $100,000 contract. Each 1/64 of a point movement represents a $15.63 change in the option premium. On June 20, 1984, the premium for a call option on the December, 1984, futures contract with a strike price of 64 closed at 1-14. The purchaser of this option would pay $1,218.75 for the right to purchase one December, 1984, T-bond futures contract at a price of $64,000.

Table 2-3. Options on Treasury bond futures Wednesday, June 20, 1984

Strike price	Calls-last			Puts-last		
	Sep 84	Dec 84	Mar 85	Sep 84	Dec 84	Mar 85
58	3-23	--	--	0-59	1-36	2-13
60	2-05	2-38	2-62	1-28	2-27	3-11
62	1-11	1-51	2-10	2-28	3-35	--
64	0-37	1-14	1-34	3-52	4-58	--
66	0-17	0-46	1-03	5-27	6-20	--
68	0-06	0-29	--	7-18	8-02	--
70	0-02	0-19	--	9-18	--	--

Calls: Volume (6/19/84) 14,792 open interest 171,738
Puts: Volume (6/19/84) 8,335 open interest 92,876

The premiums on put and call options depend on a number of factors; the theoretical foundations of the determination of futures options prices are developed in Chapter III. One factor is the strike price relative to the current futures price. For call options, the larger is the current futures price relative to the call's strike price, the larger will be the premium, other things equal. For put options, the larger the current futures price relative to its strike price, the lower will be its premium, other things equal. For a call (put) option, a futures price greater (less) than the strike price would enable the holder of the option to purchase (sell) the futures contract at a price which is less (greater) than its current market price. The larger is this gap, the more valuable is this option; hence, the larger will be premium commanded for this right.[4] On June 20, the premium on a December, 1984, call with a strike price of 60 was 3-28, while the premium for the same maturity call with a strike price of 64 was only 1-14. The opposite is true for the put; the premium for a strike price of 60 was 2-27, while it was 4-58 for a strike price of 64.

The premiums on options with the same strike price also vary by the length of time until maturity of the underlying futures contract. For both put and call options, the longer the time to maturity, the larger will be the premium, other things equal.

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[4] A call option is in-the-money when the current futures price is greater than the strike price. When the futures price is less than the strike price, the call is out-of-the-money. The opposite relationships exist for put options.

For example, the premium for a December, 1984, call with a strike price of 60 was 2-38 on June 20, while the March, 1985, call premium was 2-62 for this strike price. For the same futures maturity dates and strike price, the respective put premiums were 2-27 and 3-11, respectively.

Daily trading limits for the option premiums are the same as for T-bond futures, which are currently two points, or $2,000; i.e., the option premium could move a maximum of two points above or below the previous day's closing price. Variable limits are also the same as for T-bond futures, currently three points, or $3,000. These variable limits establish a larger range of possible price movements when the price of the option has closed up or down the limit for three consecutive days in three or more contract months during a business year.

The buyer of a T-bond futures option may exercise the option on any business day prior to expiration by giving notice to the Clearing Corporation of the Board of Trade by 8:00 p.m. The Clearing Corporation then assigns the notice to an option seller. The seller of the option does not know prior to notice whether the option will be exercised. If an option is exercised, the Clearing Corporation establishes a futures position for each buyer who has exercised the option and an opposite futures position for the seller who is given notice of exercise. The positions are established at the exercise price of the option before opening of trading on the following business day. If a call option with an exercise price of 62 is exercised, the holder of the option purchases the futures contract at 62 and the options writer sells the futures contract at the same price.

T-bond futures options cease trading on the Friday preceding by at least five business days the first notice day for the corresponding T-bond futures contract. For the June, 1984, futures contract, the first notice day was May 31, 1984, the last trading day for options on this contract was May 18, and the expiration date for options was May 19. The date of expiration for unexercised options is the first Saturday following the last day of trading.

Unlike T-bond futures contracts, there are no margin requirements for purchasers of futures options, because the buyer's risk is limited to the amount paid for the option premium. Writers of both put and call options must post performance margins. Minimum performance margins are established by the Board of Trade, and brokerage firms often require margin deposits which exceed these minimums. Writers of options are also subject to maintenance margins in addition to the initial performance margin. Both writers and buyers of T-bond futures options must pay commission charges to brokerage firms which trade these contracts and these fees are established by trading firms.

The Size of the Treasury Bond Futures Options Market

Since the inception of trading in T-bond futures options in October, 1982, there has been a rapid and sustained growth in the volume of trading and the number of contracts outstanding. Although options are currently traded for only three T-bond futures contracts, the number of call options outstanding is approaching the total open interest on all T-bond futures contracts.[5] On July 14, 1983, open interest on both types of options was 62,887, by November 16, 1983, the number of outstanding contracts reached 92,476, and by June 20, 1984, the combined open interest was 264,614 contracts. In less than one year, the outstanding contracts more than quadrupled. By June, 1984, total open interest on options contracts exceeded the number of T-bond futures contracts outstanding.

Daily trading of options has also increased quite rapidly. On July 14, 1983, the total volume of contracts traded was 12,031; on November 16, 1983, a new record volume of trades was set of 19,890 trades; and the volume for June 20, 1984, was 23,127 contracts (see Table 2-3 for data on the June 20, 1984, futures).

Thus, the T-bond futures option market has become quite liquid, at least for futures contracts with maturities up to nine months. Trading in call options has consistently been greater than for put options, both in terms of the daily volume of trades and in the number of contracts outstanding. The volume of trading in calls on June 19, 1984, was 1.77 times as large as for puts, and, on June 20, the open interest for calls was 1.85 times as large as for puts.

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[5]On June 20, 1984, there were futures contracts traded for eleven different maturity dates from June, 1984, through December, 1968. Options were traded on only the first three of these contracts -- September and December, 1984, and March, 1985.

CHAPTER III. HEDGING AND OPTIONS THEORY

In this chapter, the theory of hedging and of options pricing will be developed. In the first section, the theoretical foundations of hedging theory will be presented. Recent studies incorporating the use of financial futures in hedging theory will also be analyzed. The applicability of the theory to hedging cash positions in long-term mortgages will be assessed. The second section will cover the theoretical development of pricing of options on futures contracts. The first part of this section will consist of the development of the theory of call options pricing and the second part will be concerned with put options pricing.

In the third section, alternative hedging strategies which involve either the use of financial futures, options on financial futures, or combinations of financial futures and options on these futures will be constructed. Discussion of the major empirical studies which have evaluated the effectiveness of alternative strategies will be presented in the last section. These strategies will also be assessed in terms of their applicability to the present study.

The Theory of Hedging

One of the major reasons for the existence of futures markets, if not the primary one, is that they facilitate hedging. It is possible to hedge using forward contracts, but organized futures markets provide a means for easily hedging by providing standardized contracts which are guaranteed by the futures exchange rather than the individual contracting parties. As Working (1962, pp. 434-436) points out, futures markets were traditionally viewed as essentially speculative markets. Their usefulness in hedging was regarded as a useful by-product. It has only been in the last thirty years that the importance of hedging to the existence of futures markets has been recognized.

Hedging is done by a variety of individuals for a variety of reasons which differ according to the circumstances of the hedger. Consequently, it is necessary to delineate the purposes for undertaking the hedge position. An operational definition of hedging is that the hedge "is a temporary substitute for a merchandising contract that is to be made later" (Working, 1962, p. 441).

Working delineates a number of alternative types of hedges, three of which are important in applications to financial markets. Selective hedging is the hedging of commodity stocks, or cash financial instruments, under a practice of hedging or not hedging, according to price expectations. Because the cash instruments are hedged when a price decline is expected, the purpose of hedging is not risk avoidance in the strict sense, but avoidance of loss. The second type of hedge used in financial markets is pure risk-avoidance hedging, where a hedge is undertaken to avoid any change in the market value of the cash position. Although Working considered this type of hedge to be unimportant in modern business practice, the advent of pronounced movements in interest rates in the recent past has increased its importance for a number of types of financial institutions.

Anticipatory hedging differs from selective and pure risk-avoidance hedging in that the latter two are undertaken primarily to protect an existing position in the cash market. An anticipatory hedge involves protecting a cash position that is expected to be taken in the future. For example, a bank which plans to borrow funds in six months might sell futures contracts in an attempt to lock in its future borrowing rate. To the extent that interest rates and futures prices move inversely, an increase in the cost of borrowing caused by higher interest rates would be offset by the gain from purchasing the futures contract at a lower price than it was originally sold.

The primary purpose of this study is to assess the effectiveness of hedging a position in the cash market. Consequently, the theory of anticipatory hedging will not be developed.

Traditional hedging theory emphasized the pure risk-avoidance potential of futures markets. It was argued that hedgers would take a position in the futures market equal to, but opposite of, their position in the cash market.[1] Thus, the hedger would hedge a cash position with a futures position of equal magnitude but of opposite sign. Traditional hedging theory arguments were based on the assumption that cash and futures instruments' prices generally move together; thus, the gain or loss on the hedged position would be less than for an unhedged position.

If the cash instrument's price at times t_1 and t_2, respectively, is M_1 and M_2, and the futures instrument's price at these times is F_1 and F_2, respectively, where $t_2 > t_1$, the value of the gains or losses from the unhedged and hedged positions are, respectively:

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[1]The material on the traditional theory of hedging is based on Ederington's 1979 article.

$$U = X_M(M_2-M_1) \tag{3-1}$$

$$H = X_M[(M_2-M_1) - (F_2-F_1)], \tag{3-2}$$

where

U = return on the unhedged position,

H = return on the hedged position,

X_M = size of the cash position.

Since the cash position would be hedged by a short sale of the futures contract, the return on the futures position would be the difference between the futures price at the time it is lifted and its initial price.

If the prices of the cash and futures instruments move together, the variance on the hedged position would be less than on the unhedged position, and the risk measured in terms of the variability of the return would be decreased. The risk reduction potential is often discussed in terms of the basis, which is defined as the difference between the prices of the futures and cash instruments at each time t, or F_t-M_t. Letting $B_t = F_t-M_t$ be the basis at time t, equation (3-2) can be rewritten as:

$$H = -X_M[(F_2-M_2) - (F_1-M_1)] = -X_M \Delta B. \tag{3-3}$$

Traditional hedging was based on the argument that changes in the basis were quite small relative to the prices of the instruments because of the possibility of making or taking delivery of the commodity.

Working (1953) criticized the pure risk-minimization assumption of the traditional hedging theory. He argued that hedging was undertaken primarily to maximize profits. Hedgers were viewed as taking positions in the futures market depending on their expectations of changes in cash-futures price relationships. Thus, a holder of a long cash position would hedge by selling futures contracts only if the basis was expected to narrow, and would not hedge if it was expected to widen.[2] Despite the fact that Working's original article was published more than thirty years ago, many articles on hedging either ignore the possibility that there can be substantial changes in the basis or include only a disclaimer that these changes might occur and consequently result in a return which is different from the hypothesized return.[3]

Changes in the basis can be especially important when a cross-hedge is being undertaken. A cross-hedge is one for which the futures contract instrument is different than the cash instrument. In the present study, only cross-hedges are considered since the cash instrument is outstanding mortgage loans and there is no futures contract traded on any mortgage contract. In this instance, the possibility of a change in the basis over the hedging period is of special significance.

Johnson (1960) and Stein (1961) applied basic portfolio theory to the theory of hedging. This application resulted in incorporating the risk minimization of the traditional hedging theory and the maximization of expected profit theory of hedging developed by Working into a unified theory of hedging. A number of alternative measures of risk might be used. Measures of risk which focus on the disutility aspects of uncertainty are the

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[2]The relationship between the change in the basis and the return on the hedged position is $\partial H/\partial \Delta B = -X_M$ from equation (3-3). Since $X_M > 0$, the return is inversely related to the change in the basis. If the basis is expected to narrow, or $E(\Delta B) < 0$, there would be an expected gain on the hedged position and the hedge would be undertaken. The opposite is true if the basis is expected to widen.

[3]Among these are articles in exchange, brokerage house, and trade publications.

probability of loss, the expected value of loss, and the variance of the expected return. The measure used in this study is the variance of the expected return.

One difference between basic portfolio theory and the portfolio model of hedging is that the cash and futures holdings are not considered to be substitutes in the portfolio model of hedging. Additionally, since cash holdings are assumed to be predetermined, any interest receipts on the cash instrument(s) holdings may also be considered to be given and, thus, have no effect on the analysis.

A seminal article on the portfolio approach to hedging was written by Ederington (1979). Ederington assumed, as did Johnson and Stein, that only one cash market instrument was being hedged. This assumption also holds in the present study. The derivation of the optimal proportion of the cash instrument to be hedged used in this study follows from Ederington's work.

Defining the return on the unhedged position, as in equation (3-1), its expected value and variance are:

$$E(U) = X_M \cdot E(M_2 - M_1), \text{ and} \qquad (3\text{-}4)$$

$$Var(U) = X_M^2 \, \sigma_M^2 \qquad (3\text{-}5)$$

where σ_M^2 is the subjective variance of the cash instrument and E is the expectations operator.

If R represents the return on the hedged position which contains cash and futures instrument holdings of X_M and X_F, respectively, the expected return and variance of the hedged position are:

$$E(R) = X_M \cdot E(M_2-M_1) + X_F \cdot E(F_2-F_1) - K(X_F) \tag{3-6}$$

$$Var(R) = X_M^2 \sigma M^2 + X_F^2 \sigma F^2 + 2X_M X_F \sigma_{MF}, \tag{3-7}$$

where $K(X_F)$ represents brokerage and other costs of undertaking futures transactions, including the cost of providing margin, and σ_M^2, σ_F^2 and σ_{MF} represent the subjective variance and covariance of the possible price changes between periods one and two.

The cash position may be completely hedged or unhedged. There is no a priori assumption concerning the extent to which the cash position is hedged. In the traditional theory, it was assumed that $X_F = -X_M$, and Working assumed that $X_F = -X_M$, or zero, depending on the expected change in the basis. It is possible that X_M and X_F have the same sign, which would mean that a long (short) position in the cash market is hedged by taking a long (short) position in the futures market.

Let $n = -X_F/X_M$ represent the proportion of the cash position which is hedged. If a long cash position is hedged by taking a short position in the futures market, X_M and X_F would have opposite signs and n would be positive; i.e., a short sale of the futures contract would be indicated by a negative value. The value of n is the number of futures contracts traded for each unit of the cash instrument held.

Substituting n into equations (3-6) and (3-7) gives

$$\begin{aligned} E(R) &= X_M \cdot E(M_2-M_1) - nX_M \cdot E(F_2-F_1) - K(X_M,n) \\ &= X_M[E(M_2-M_1) - n \cdot E(F_2-F_1)] - K(X_M,n) \end{aligned} \tag{3-8}$$

$$\begin{aligned} Var(R) &= X_M^2 \sigma M^2 + n^2 X_M^2 \sigma_F^2 - 2nX_M^2 \sigma_{MF}^2 \\ &= X_M^2(\sigma_M^2 + n^2\sigma_F^2 - 2n\,\sigma_{MF}). \end{aligned} \tag{3-9}$$

Letting the expected change in basis be $E(\Delta B) = E[(F_2-M_2) - (F_1-M_1)]$, the expected return on the hedged position is

$$E(R) = X_M[(1-n)\cdot E(\Delta M) - nE(\Delta B)] - K(X_{M,N}), \tag{3-10}$$

where $E(\Delta M) = E(M_2-M_1)$ is the expected change in the price of one unit of the cash instrument.

If the expected change in the basis is zero, the expected gain or loss is reduced as n approaches one, as in the traditional theory. It can also be seen that changes in the basis can add to, or reduce, the return that would have been expected on the unhedged position where $E(U) = X_M(\Delta M)$.

Because the size of the holding of the cash instrument is assumed to be constant, the effect of a change in the proportion of the cash position hedged on the variance of the return is

$$\frac{\partial Var(R)}{\partial n} = X_M^{\;2}(2n\sigma_F^{\;2} - 2\sigma_{MF}) = 0, \tag{3-11}$$

and the risk minimizing hedge ratio is

$$n^* = \sigma_{MF}/\sigma_F^{\;2}. \tag{3-12}$$

There is no a priori restriction on the sign of n^*. If the prices of the cash and futures instruments move in opposite directions $\sigma_{MF} < 0$, the risk minimizing hedger would then take the same position in both markets, i.e., a holder of a cash position would take a long position in the futures market. If $n^* > 1$, each unit of the cash instrument would be hedged with a short position of more than one futures contract. The only case for which the one-to-one hedge ratio of the traditional theory would be optimal would be when cash and futures price movements are equal.

A measure of the effectiveness of the hedge can also be derived using the portfolio approach. If the effectiveness of the hedge is defined as the risk reduction potential of the position, one can determine its effectiveness by comparing the risk on the unhedged position with the minimum risk that can be obtained on the hedged position. The measure of hedging effectiveness used in this study is

$$e = 1 - \frac{Var(R^*)}{Var\ (U)} \tag{3-13}$$

where Var(R*) represents the minimum variance of the hedged position. Substituting equation (3-12) for the risk minimizing hedge ratio into equation (3-9) gives

$$\begin{aligned} Var(R^*) &= X_M^2(\sigma_M^2 + \sigma_{MF}^2/\sigma_F^2 - 2\sigma_{MF}^2/\sigma_F^2) \\ &= X_M^2\sigma^2_M(1-\sigma^2_{MF}/\sigma^2_M \cdot \sigma^2_F). \end{aligned} \tag{3-14}$$

Substituting (3-14) into equation (3-13) yields

$$e=1 - (1-\sigma^2_{MF}/\sigma^2_M \cdot \sigma^2_F) = \sigma^2_{MF}/\sigma^2_F \cdot \sigma^2_M = \rho^2, \tag{3-15}$$

which is the population coefficient of determination between the change in the cash instrument's price and the change in the future's price. If the hedge is perfect, or Var(R*) is zero, the value of e will be one. As the percentage variation in the hedged position is reduced, e increases. Thus, e measures the percentage reduction in risk resulting from the hedge.[4]

[4]Regressing the change in the cash instruments' price on the future's price provides an estimate of the risk minimizing hedge ratio n*, since $n^* = S_{MF}/S^2_F$, where S_{MF} and S^2_F are the sample covariance of the changes in the cash and futures prices and the variance of the change in the futures price, respectively. The effectiveness of the hedge is estimated by the sample coefficient of determination r^2.

Options Pricing Theory

The theoretical foundation of the theory of pricing of options on futures contracts is derived from option pricing models for corporate stocks. The futures option pricing model used in this study was developed by Black (1976) and is based on the Black-Scholes (1973) stock option pricing model. In the Black model, the price of a call option which can only be exercised at maturity (a European option) is derived from the assumption that a riskless hedged position consisting of a short position in the futures contract and a long position in the futures option can be created.[5]

The major assumptions of the Black model are:

(i) The markets for futures and options are fiction less; e.g., there are no restrictions on short sales, no transactions costs and no taxes;

(ii) The risk-free rate of interest is known and constant over the life of the option;

(iii) The fractional change in the futures price over any interval is distributed log-normally with a known variance rate equal to σ^2 which is constant over the life of the option;

(iv) The option is "European;" that is, it can only be exercised at maturity.

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[5]This section is based on Black and Scholes (1973), Black (1976), and Jarrow and Rudd (1983).

The terms used in the derivation are:

C = Value of the call option;

F = Price of the futures contract;

E = Exercise price of the call option;

r = Riskless rate of interest;

σ^2 = Variance rate of the fractional change in the futures price;

t = Time;

T = Time of expiration of the option.

Under these assumptions, the value of the option will depend only on the price of the futures contract, on the time to expiration, and on variables that can be taken to be known constants. It is then possible to create a hedged position consisting of a long position in the option and a short position in the futures contract whose value will depend only on time and the values of known constants, but not on the price of the futures contract. In order for the hedge to be riskless, the hedger must be able to adjust the position continuously changing the ratio of options to futures held. The value of the option expressed as a function of the futures price F and time t is C(F,t). The number of options which must be sold short against one futures contract held is $\frac{1}{\partial C/\partial F}$.[6] Since futures

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[6]The Black model assumes that the hedge can be continuously adjusted at zero cost. In order for the hedge to be riskless, the change in the value of the position must be zero; thus, the hedge ratio is $H = \partial C/\partial F$, since an instantaneous unit change in the stock price causes a change in the price of the call option of this magnitude. There is no theoretical justification for assuming that H is constant at alternative values of F, so, as the price of the stock changes, the hedge ratio must be adjusted to keep H equal to $\partial C/\partial F$.

contracts are settled daily with gains being added to the trader's account and losses deducted from his/her account, the value of the futures contract is reset to zero each day. The value of the equity of the hedged position is then just the value of the option.

The change in the value of the hedged position over the time interval Δt is

$$\Delta C - \partial C/\partial F \cdot \Delta F. \tag{3-16}$$

Assuming that the short position can be continuously adjusted, stochastic calculus can be used to expand ΔC, which is $C(F + \Delta F, t+\Delta t) - C(F,t)$ as follows:

$$\Delta C = \frac{\partial C}{\partial F} \cdot \Delta F + \frac{1}{2} \cdot \frac{\partial^2 C}{\partial F \cdot \partial t} \cdot \sigma^2 \cdot F^2 \Delta t + \frac{\partial C}{\partial t} \cdot \Delta t. \tag{3-17}$$

Substituting from equation (3-17) into (3-16), the change in the value of the hedged position is:

$$\Delta C - \frac{\partial C}{\partial F} \cdot \Delta F = \frac{\partial C}{\partial F} \cdot \Delta F + \frac{1}{2} \frac{\partial^2 C}{\partial F \partial t} \cdot \sigma^2 F^2 \Delta t + \frac{\partial C}{\partial t} \cdot \Delta t - \frac{\partial C}{\partial F} \Delta F$$

$$= \frac{1}{2} \frac{\partial^2 C}{\partial F \partial t} \cdot \sigma^2 F^2 \Delta t + \frac{\partial C}{\partial t} \cdot \Delta t. \tag{3-18}$$

Since the change in the value of the equity must equal the value of the equity times $r\Delta t$ for a riskless hedge, we have:

$$\frac{1}{2} \frac{\partial^2 C}{\partial F \partial t} \cdot \sigma^2 F^2 \Delta t + \frac{\partial C}{\partial t} \cdot \Delta t = Cr\Delta t. \tag{3-19}$$

Cancelling out Δt from each side of this equation and solving for $\partial C/\partial t$ gives the differential equation

$$\frac{\partial C}{\partial t} = Cr - \frac{1}{2} \frac{\partial^2 C}{\partial F \partial t} \cdot \sigma^2 F^2. \tag{3-20}$$

At the time of expiration, the value of the call option on the futures contract is the difference between the price of the futures contract at this time and the option's exercise price if the difference is positive. If the futures price is less than the exercise price at expiration, the call option will have a value of zero. If the futures price is greater than the exercise price, the holder of the option could purchase the futures contract at a price below its current market price resulting in a gain. If the futures price were less than the exercise price, the holder would allow the option to expire unexercised; thus, its value would be zero. This relationship can be written as:

$$\begin{aligned} C(F,T) &= F_T - E, \quad F_T > E \\ &= 0 \quad\;\;\;\;\;\;\;, \quad F_T \leq E. \end{aligned} \tag{3-21}$$

Equation (3-21) is the main boundary condition for the value of the call option.[7] Solving equations (3-20) and (3-21) for C gives the formula for the value of a call option on a futures contract, which is:

$$C(F,t) = e^{-r\tau}\,[F\cdot N(d_1) - E\cdot N(d_2)], \tag{3-22}$$

where $d_1 = [\ln(F/E) + 1/2\cdot\sigma^2\tau]/\sigma\sqrt{\tau}$;

$d_2 = [\ln(F/E) - 1/2\cdot\sigma^2\tau]/\sigma\sqrt{\tau} = d_1 - \sigma\sqrt{\tau}$;

$\tau = (T-t)$, which is the time to expiration;

$N(d_i)$ is the cumulative normal density function for d_i.

The price of the call option then depends on its time to maturity, its exercise price, the riskless rate of interest, and the futures price and

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[7]A second boundary condition necessary to make the solution to equations (3-20) and (3-21) unique is that $C(0,t) = 0$.

the variance rate of the fractional change in this price, or the volatility of the futures contract's price.

The Black model was derived under the assumption that the call option is a European option, which does not have an early exercise right. There has been a debate in the stock options literature on the early exercise right for American options. Jarrow and Rudd (1983, p. 63) demonstrated that American options on nondividend-paying securities will never be exercised early. Since there are no dividends paid on future contracts, the early exercise right does not increase the value of the option.

The implications of this model can be examined by evaluating the effects of a change in the value of each of the model's parameters on the option's price.[8] The partial derivatives of the call price for changes of each of the parameters are:

$$\frac{\partial C}{\partial F} = e^{-rT} \cdot N(d_1) > 0 \tag{3-23}$$

$$\frac{\partial C}{\partial E} = -e^{-rT} \cdot N(d_2) < 0 \tag{3-24}$$

$$\frac{\partial C}{\partial r} = -Te^{-rT}[F \cdot N(d_1) - E \cdot N(d_2)] < 0 \tag{3-25}$$

$$\frac{\partial C}{\partial \sigma} = e^{-rT} T \cdot F \cdot N'(d_1) > 0 \tag{3-26}$$

$$\frac{\partial C}{\partial T} = \frac{Te^{-rT} F \cdot \sigma}{2} N'(d_1) + e^{-rT} ErN(d_2) \gtreqless 0, \tag{3-27}$$

where $N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-d_1{}^2/2}$.

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[8]For the derivation of the effects of changes in each of the model's parameters on the value of a call option on a corporate stock, see Jarrow and Rudd (1983), pages 119-120. The derivations in this section are based on Jarrow and Rudd's work with the necessary modifications to account for the difference between stock and futures options. Goodman (1983)

For a call option, the effect of a change in the futures price on the option's price is unambiguous. As is derived in equation (3-23), the value of the call option increases when the futures price increases, because the intrinsic value of the option increases when the futures price increases. The higher (lower) is the exercise price E of the option, the smaller (larger) is the value of the option, as should be obvious, because the buyer is paying for the right to purchase the futures instrument at a higher price; consequently, the right is worth less.

The effect of a change in interest rates on the option's price is not as obvious as in the previous two cases. To understand this effect, consider a riskless world. The option buyer would be paying an amount equal to the present discounted value of the difference between the value of the futures contract at expiration and the option's exercise price. An increase in the riskless rate of interest would cause this present value, which is the price of the option, to decline. However, since the futures price at the option's expiration date and the exercise price are expected to be fairly close, interest rate changes have only a slight effect on the price of the option. This derivation assumes that the price of the futures contract is unaffected by changes in the riskless rate of interest, or $\partial F/\partial r = 0$. Substantial movements in the prices of debt instruments will occur because of changes in interest rates. As a rule, interest rate increases will cause a fall in the price of futures contracts whose underlying instrument is a debt instrument, such as Treasury bond futures. This

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discusses the effects of changes in the model's parameters on the price of a futures option but does not derive them mathematically.

effect will reinforce the negative effect of the riskless interest rate on the opportunity cost of the call premium.

The parameter σ measures the volatility of the futures price. As the futures contract price becomes more volatile, the value of the call option increases. This is because a call option has no downside risk, since, no matter how low the futures price falls, the option's minimum value is zero, and increasing volatility increases the probability that the futures price will be greater than the exercise price at expiration of the option. An increase in the probability that the option will be in-the-money causes an increase in the value of a call option.

For a call option, an increase in the time to expiration decreases the present value of the difference between the futures price and the exercise price paid by the purchaser at the time of expiration. This causes a decrease in the value of the call option. However, increasing the time to maturity of the option increases the likelihood of favorable outcomes for the buyer of the option. Hence, since the two effects are opposite in influence, the effect of a change in the time to expiration of the call option is ambiguous.

The valuation of a European put option follows directly from the Black formula for a call option and the application of the put-call parity equation. The value of a put option on a futures contract is:

$$P = C-e^{-rT} \cdot (F-E), \tag{3-28}$$

where P is the price of the put option.

The derivation of the put option price is based on the analysis of the cash flows from the arbitrage portfolio. Assume that a portfolio is constructed consisting of writing a call option, purchasing a put option, and purchasing the futures contract at time t. Further, assume that all of the instruments in the portfolio have the same expiration date, the put and call options have the same exercise price, and that all the assumptions of the Black call option model hold. Let the prices of the put and call options at time t be P_t and C_t, respectively, the exercise price for the options be E, and the prices of the futures contract at the initiation of hedge and at expiration of the contracts be F_t and F_T, respectively.

The cash flows from the arbitrage portfolio are given in Table 3-1. In order to derive these cash flows, first consider the call option. This option will be exercised only if the futures price at T is greater than the option's exercise price, or $F_T>E$; so, the loss to the writer of the option is F_T-E, or zero, whichever is greater. The lower bound of zero occurs when $F_T<E$ and the option expires unexercised. When the option is exercised, the writer of the call option would offset his/her position by delivering the futures contract which was purchased at time t. The put option would be exercised only if the futures price is less than the exercise price at expiration, or when $F_T<E$, so the gain on the option would be $E-F_T$ or zero. When the put is exercised, the purchaser of the put would deliver the futures contract which was initially purchased. If $F_T>E$, the put option would expire unexercised.

It can be seen from Table 3-1 that the cash flow at the expiration of the hedge does not depend on whether the futures price is greater, or less, than the exercise price of the options. The value of the arbitrage portfolio at the options' expiration is known at the initiation of the position because both the exercise price E and the purchase price of the futures contract F_t are known.

Table 3-1. Cash flows from the arbitrage portfolio

Position	Opening transaction	Closing transaction	
		$F_T > E$	$F_T < E$
Write a call	C_t	$E-F_T$	0
Buy a put	$-P_t$	0	$E-F_T$
Buy a futures contract	0	F_T-F_t	F_T-F_t
Total	C_t-P_t	$E-F_t$	$E-F_t$

The initial value of the arbitrage portfolio must be equal to zero if there are no riskless profits to be made, so, the present discounted value of the cash flows must be zero. Letting r equal the riskless rate of interest and τ-t be the time to expiration of the option, the present discounted value of the cash flow is:

$$0 = C_t - P_t + e^{-r\tau}(E-F_t), \tag{3-29}$$

which, through rearrangement of terms and dropping of the time subscript, is the put-call parity equation of (3-28).

The put-call parity equation can then be used to solve for the price of a put option on the same futures contract and with the same exercise price and expiration date as the call option by using the Black call option model and the parameters r and τ of the call option model.

The effect of a change in any of the parameters of the model on the put option price can be derived from the impact on the call option price and the put-call parity equation. A change in the price of the underlying futures contract will cause an inverse change in the price of the put option. An increase (decrease) in the futures price will mean that the buyer of the put option will have to pay a higher (lower) price to purchase the contract in order to offset the sale of the contract (the initial put). A higher (lower) exercise price will cause the price of the put option to increase (decrease). A higher (lower) exercise price gives the buyer of the put the right to sell the option at a higher (lower) price. Thus, the value of this right will increase (decrease) with an increase (decrease) in the exercise price.

The value of a put option will decrease (increase) when the rate of interest increases (decreases). There are two effects on the value of the put when the rate of interest changes. The first effect is the change in the call price which is always negative, and the second is the effect on the present value of the difference between the futures price and the exercise price. If the futures price is less than the put's exercise price, a decrease (an increase) in the rate of interest will cause a decrease (an increase) in the present value of this difference, and this will reinforce the negative effect on the put's price from a change in the call option's price. If the futures price is greater than the exercise price of the put option, the effect on the put's prices is less obvious. In this case, an increase in the risk-free rate of interest will cause the value of the term $[-e^{-rT}(F-E)]$ to increase, but this will never be large enough to offset the decline in the put price caused by a decline in the call option's price. Thus, there is an inverse relationship between the risk-free rate of interest and the price of a put option. It is assumed in this derivation that the price of the futures contract is independent of the risk-free rate of interest ($\partial F/\partial r = 0$).

As is the case for the price of a call option, a change in the volatility of the futures price (measured by σ) will cause the put option's price to change in the same direction. This is because there is no downside risk on the option, and, consequently, an increase (a decrease) in the volatility will increase (decrease) the probability that the futures price will be less than the exercise price at the expiration of the option contract. An increase (a decrease) in the probability that the put option will be in-the-money will cause an increase (a decrease) in the value of the option.

A change in the time to expiration of the put option will have two effects on its value. Increasing (decreasing) the time to expiration will decrease (increase) the present discounted value of the difference between the exercise price and the futures price, if positive. However, increasing (decreasing) the time to expiration increases (decreases) the likelihood of a favorable outcome. Hence, as the time to expiration changes, the effect on the put's value will depend on the relative magnitude of these effects.

Hedging with Futures and Futures Options

In this section, a number of alternative strategies which can be used to hedge a cash position are presented. These strategies involve using financial futures contracts, options on these futures contracts, and combinations of futures and options contracts. Additionally, a strategy of using futures contracts and stop orders on the contracts will also be evaluated. It will be assumed throughout the analysis that the strategies consist of equal size futures and option positions. For example, if the two alternative strategies being considered are selling futures contracts or buying put options on futures contracts, the number of futures contracts sold will be assumed to be equal to the number of put options purchased. It is a straightforward extension of the present analysis to evaluate the results of strategies which use alternative futures/options weighting systems. It will also be assumed that the hedging position will be weighted by the risk minimizing hedge ratio n* derived previously in this chapter. If put options on futures contracts are the hedging instrument, for each unit of the cash instrument held, n* units of put options will be purchased.

The alternative hedging strategies evaluated are: selling futures contracts, selling futures contracts and simultaneously purchasing call options on the same futures contracts, purchasing put options on futures contracts, and selling futures contracts and placing stop orders on these contracts.

The return to an unhedged position in the cash instrument was given in equation (3-1) as $U = X_M(M_T-M_t)$, where the subscripts t and T represent the times at which the hedge is initiated and lifted, respectively. The risk minimizing hedge ratio derived in equation (3-12), $n^* = \frac{\Delta M}{\Delta F}$, results in an expected return on the hedged portfolio of:

$$E(R) = E[X_M(\Delta M - n^* F)] = E[X_M(\Delta M - \Delta M/\Delta F \cdot \Delta F)] = 0,$$

where $\Delta M = M_T - M_t$ and

$\Delta F = F_T - F_t$.

The return on the hedged portfolio using futures contracts is presented in Figure 3-1, where the initial futures price is F_t and its price when the hedge is lifted is F_T. The curve for the return on the cash holding is weighted by $1/n^*$ in order to make the analysis consistent; i.e., the return is for $1/n^*$ units of the cash instrument and one unit of the futures instrument. In the figure, the slopes of the return lines will be of equal absolute size, since, for each one unit change in the price of the futures contract, the value of $1/n^*$ units of the cash instrument will be $n^* \cdot 1/n^* = 1$. The return on the sale of a futures contract is positive when $F_T < F_t$, because the seller of the contract can repurchase it at a lower price than its initial selling price.

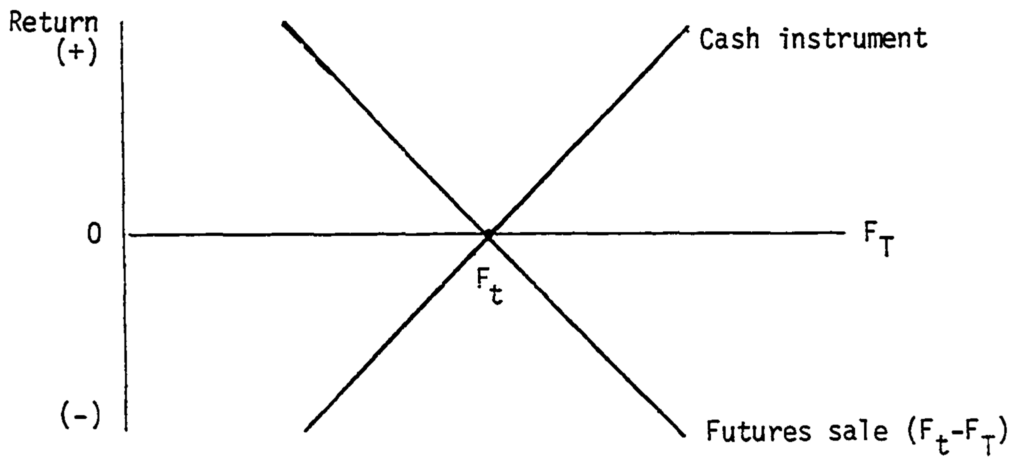

Figure 3-1. Return from cash and futures position

The total return for the hedged position is then zero at each alternative price of the futures contract. It has been implicitly assumed that $n^* > 0$ in this derivation, so that a cash position is hedged with a short position in the futures instrument.

An advantage of using futures options as hedging instruments is that the downside risk of a movement in the cash instrument's price is eliminated while the upside risk, or a favorable movement, is not eliminated. The purchase of an option can be considered to be a form of insurance, a favorable cash instrument price movement will result in a gain, while an unfavorable movement is insured against by holding the option. For example, a strategy of buying a futures contract and purchasing a put option on this contract at an exercise price equal to the purchase price of the futures contract will eliminate the possibility of a loss from holding the futures contract. If the price of the futures contract were to fall, the potential loss on the contract could be offset by exercising the put, thus selling the futures contract at its initial purchase price. The price paid for the put option, or the option's premium, is the price paid for the insurance against loss.

A hedge which incorporates the use of futures options is the sale of a futures contract and the simultaneous purchase of a call option on the contract. The cash instrument is then hedged by a futures/options position. Assume that the exercise price of the call option is equal to the initial (selling) price of the futures contract and one call option is purchased for each futures contract sold. The return from this strategy is illustrated in Figure 3-2. C_t, as previously defined, is the price of one call option. Since the option is purchased, C_t has a negative value for the return on the option.

The return from the futures sale is F_t-F_T, where F_T, as before, is the price of the futures contract at the time the position is lifted. If

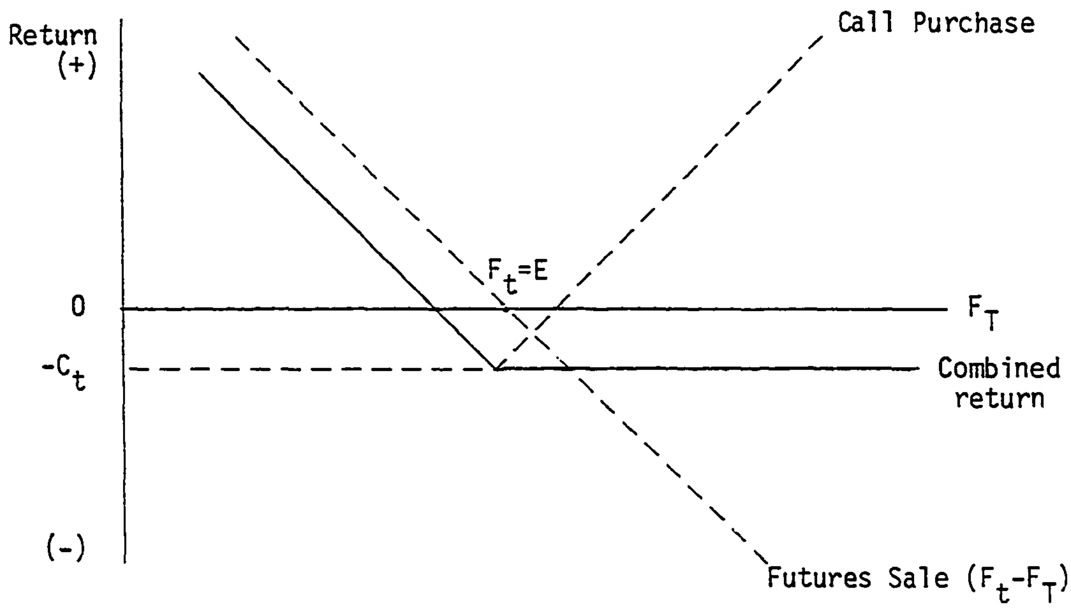

Figure 3-2. Return from a futures sale and call option purchase

the price of the futures contract at this time is greater than the exercise price of the option, the option would be exercised and the futures contract repurchased at the exercise price of E; thus, if $F_T > F_t$, the return is F_t-E-C_t. If the closing futures price is less than the option's exercise price, the position would be closed out by repurchasing the futures contract at F_T and allowing the option to expire unexercised. The return from the futures/options position is then:

$$R_1 = F_t-E-C_t, \quad F_T > E$$
$$= F_t-F_T-C, \quad F_T < E. \tag{3-31}$$

Since it is assumed that the exercise price of the option is equal to the initial futures price, the minimum return on this position is $-C_t$, or the price of the call option.[9] Equation (3-31) is also the return to the futures/options position when the initial futures price and the option's call price are not equal because the option would only be exercised if the closing futures price were greater than the option's exercise price.

The sell futures/buy call options strategy can be replicated by an options only strategy. In this case, the strategy would be to purchase put options on the futures contract. Again, assume that the exercise price of the put option on the futures contract equals the initial price of the futures contract. If the futures price at the end of the hedge period is greater than the options exercise price, or $F_T>E$, the option will not be exercised, because the selling price E is less than the repurchase price F_T, thus resulting in a loss. If $F_T<E$, the option would be exercised resulting in a gain of $E-F_T-P$. Because it is assumed that the initial futures price and the option's exercise price are equal, the return to the put option purchase is:

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[9]Assume that the initial price of the futures contract is 70, one futures contract is sold at this price, and one futures call option with an exercise price of 70 is purchased at a price of 3. If the futures price at the end of the hedging period were 75, the option would be exercised ($F_T>E$) and the return on the position would be $F_t-E-C_t = -C_t = 70-70-3=-3$.

If the futures price were to fall to 65, the call option would not be exercised; instead, the position would be closed out by purchasing the futures contract at this price. In this case, the return on the position would be $F_t-F_T-C_t = 70-65-3 = 2$. It is also obvious that, as the futures price at the termination of the hedged period decreases, the return to the position increases; i.e., if $F_T = 60$, the return is 70-60-3 = 7. The options price of 3 is the price paid to eliminate the possibility of loss on the short sale of futures contract.

$$R_2 = -P_t \qquad ,F_T > E$$
$$= E-F_T-P_t \qquad ,F_T < E, \qquad (3\text{-}32)$$

where P_t is the price of the put option. Equation (3-32) holds no matter what the initial futures price is, because there is no initial futures market transaction.[10]

The return from the sale of a put option on a futures contract is illustrated in Figure 3-3. Comparing Figures 3-2 and 3-3, it is apparent that the sell futures/buy calls strategy is replicated by the purchase of put options if the exercise prices of the call and put options contracts are equal. The returns from these two strategies under the assumption of equal exercise prices will differ only by the difference between the prices of the put and call options, or P_t-C_t.

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[10]Assume that the same numerical values hold as in the example of footnote eleven where F_t = 70 and E = 70. Also, assume that the put option's price is 3. If the price of the futures contract at the termination of the hedge is 75, the option would not be exercised because, if it were, the holder of the option would sell the futures contract at 70 and close out the position by repurchasing the contract for 75. Thus, when F_T = 75, the option would not be exercised, and the return on the position would be the cost of purchasing the put, or -3.

If the terminal futures price were 65, the holder of the option would exercise it, selling the futures contract at 70, repurchasing it at 65, and receiving a return of $E-F_T-P_t$ = 70-65-3 = 2. The put options position is then exactly replicated by the futures sale/call options purchase position when the price of the put and call options are equal and they have the same exercise price. This can easily be seen by comparing this example with the example in footnote eleven.

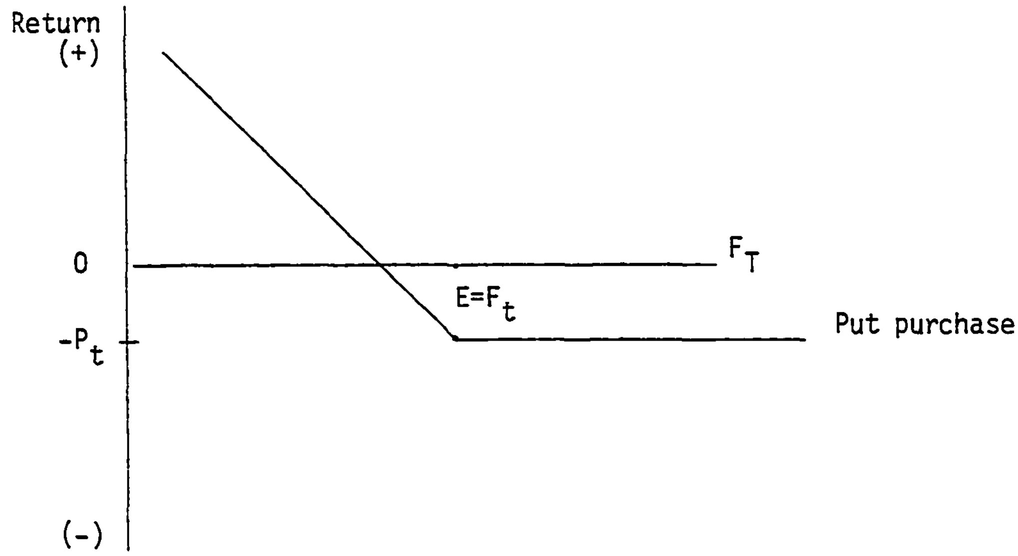

Figure 3-3. Return from buying a put option

The returns to hedging 1/n* units of the cash instrument alternatively with the purchase of a put option on a futures contract, or the sale of a futures contract and the purchase of a call option on the same contract can be derived using the relationships for each strategy. Assume that the prices of the put and call options are equal, or $C_t=P_t$, and that the exercise prices of the options are also equal. The return to the cash instrument hedged with either a put option or a call option and a futures sale is:

$$R_3 = \frac{1}{n^*} \cdot \Delta M - \Delta F - P_t = -P_t, \qquad F_T < E$$

$$= \frac{1}{n^*} \Delta M - P_t \qquad\qquad F_T > E$$

where $P_t = C_t$, $\Delta M = M_T - M_t$, and $\Delta F = F_T - F_t$.

The return from these positions is illustrated in Figure 3-4.

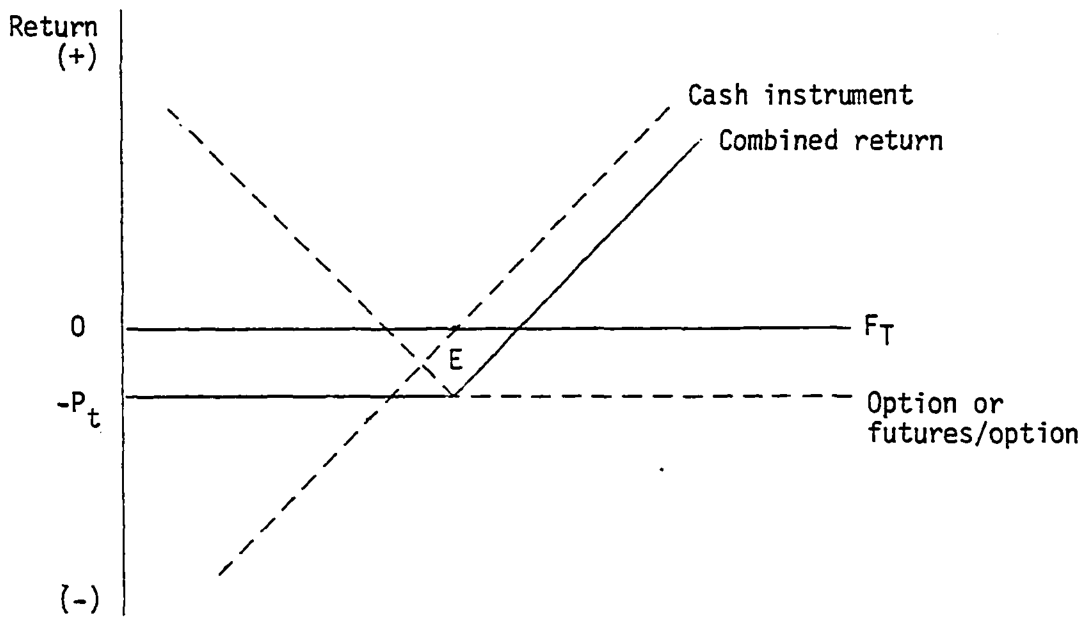

Figure 3-4. Return from hedging with put option or call option/futures sale

Figure 3-4 and equation (3-33) provide evidence of the advantage of using options in the hedging strategy. When a cash position is hedged with the sale of a futures contract, assuming $n^* > 0$, both the possibility of a loss and of a gain is reduced. Using options in the hedging position reduces the possibility of a loss when the cash instrument's price falls, but does not eliminate the possibility of a gain when the cash instrument's price rises. The price paid for the possibility of the gain is the option's price, or its premium.

The final hedging strategy considered in this study involves using futures contracts and stop orders on these contracts. Because they are most commonly used to limit potential losses, the stop orders are generally referred to as stop loss orders. A stop loss order establishes the maximum loss that a holder of a position is willing to incur. If the holder of a short position for one T-bond futures contract initially sells the contract for 75, or $75,000, and places a stop order at 78, the position will be offset if the futures price reaches this point at any time. Thus, the holder has limited his/her potential loss to $3,000, since the futures contract would be purchased for $78,000.[11]

The holder of a cash instrument could then elect to hedge the position by selling futures contracts and placing stop orders on the contracts. Since the futures contracts are sold, the stop orders would be placed at a price higher than the original futures selling price. The price at which a stop order is placed will depend on the maximum loss the hedger is willing to incur on a futures transaction. The advantage of using this strategy instead of a straight sale of the futures contract is that favorable movements of the cash instrument's price are not offset by unfavorable movements of the futures contract's price except for the difference between the price at which the stop order is executed and the original selling price of the contract.[12] Unfavorable movements of the cash

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[11] Loosigian (1980) provides an analysis of the use of stop orders for alternative futures transactions on pages 117-123.

[12] Assume that $n^*=1/2$, two units of the cash instrument are purchased at time t for $M_t=60$ per unit, and are hedged by selling one futures contract at $F_t=70$. A stop order is placed on the futures contract at a price of $F_L=73$. If the cash instrument's price increases to 62 and the futures price increases to 74 at time $t+i<T$, the return on the position would be:

$$1/n^* \cdot (M_{t+i}-M_t) + (F_t-F_L) = 2(62-60) - (70-73) = 1.$$

If the price of the cash and futures instruments would decline to 58 and 66, respectively, the return would be 2(58-60) + (70-66) = 0.

instrument's price are still protected against by the hedge when the stop order is not executed.

The futures sale/stop order strategy is an alternative to either the futures/call options or put options strategies discussed above, in that the strategy limits the loss on unfavorable cash instrument price movements, but not the gain on favorable cash instrument price movements. The advantage of using stop orders instead of futures options is that there is no cost involved in executing the order because the fees for the futures transactions are charged on a roundturn basis. The cost of using options on the hedging position is the price paid for the put or call option.

There is, however, a potentially serious disadvantage in using stop orders because the orders could be executed when, in retrospect, they should not have been. Suppose that a hedge is initiated by selling futures contracts and placing stop orders on the contracts. During the period the cash and futures instruments' prices increase sufficiently so that the stop orders are executed, but these prices subsequently fall during the hedge period. The loss on the cash instrument consequently is not offset by a gain on the futures contracts, but is increased by the loss on the futures contracts.[13] The use of stop orders then introduces

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[13]Using the initial values of the examples in footnote twelve, let the time $t+i<T$ be $M_{t+i} = 61.5$ and $F_{t+i} = 73$. The futures position would then be closed out at this price. If the cash and futures prices subsequently fall and the cash price at the termination of the hedge is M_T=56, the return on the position would be:

$$2(56-60) - (73-70) = -8-3 + -11.$$

This would be a larger loss than would have been sustained on the unhedged position.

the possibility that the futures position will be offset when, ex post, it should not have been.

Empirical Studies of Futures and Options Hedging Strategies

A number of recent studies have empirically tested the effectiveness of various hedging strategies using futures contracts and/or options on futures contracts. Some of these have focused on the use of financial futures in hedging cash positions in mortgage loans, or positions in mortgage loan commitments which have not been taken down. Others have evaluated the use of options in hedging cash positions. To this date, however, there have not been any empirical studies which evaluate both futures and futures options strategies for hedging mortgage loan positions. Those empirical studies which are most directly applicable to the present analysis are discussed in this section.

Ederington (1979) estimated the effectiveness of the GNMA CDR and Treasury bills futures markets in reducing the risk associated with holding cash positions in these instruments and estimated the effect on expected returns for the hedged positions. Although this study did not directly apply the risk minimization analysis to mortgage lending nor to the use of options on futures, it is significant in that it was the first study to use the techniques employed in the present research. Ederington found that the risk minimizing hedge ratio and the efficiency of the hedge varied according to the length of the hedge and the maturity of the contract used in the hedge. This was true for both hedges using the GNMA and

Treasury bills contracts. It was also true that longer hedges had greater efficiency than did short-term hedges.

Starleaf and Langley (1983) estimated the risk minimizing hedge ratio and its effectiveness when the cash instrument was mortgage loans and the hedging instrument was either Treasury bond or GNMA CDR futures contracts. Three alternative hedge lengths were used -- six, twelve, and eighteen months -- and, in each case, the hedging instrument was the nearby futures contract.[14] The estimates of the risk minimizing hedge ratio using the T-bond futures contract ranged from .596 to .680 and its effectiveness ranged from .46 to .59. For each of the hedging periods selected, it was found that GNMA CDR futures contracts were a less effective instrument for hedging the cash instrument than were T-bond futures.

Losey (1981) evaluated a strategy of hedging conventional mortgages using short positions in GNMA CDR futures. He assumed that conventional mortgages were originated on a continuing basis and were hedged on a quarterly basis using GNMA short positions in contracts with delivery dates one year from the date of initiation of the hedge. The position was then maintained for one year. Losey did not use a weighted hedge strategy; instead he analyzed the changes in the yields on conventional mortgage loans and GNMA CDRs. The justification for the use of changes in yields rather than changes in the instruments' prices was that the prices of seasoned

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[14]It was assumed that the cash instrument being hedged was an eight percent, thirty-year, monthly payment mortgage which would be prepaid in the twelfth year. The reason this instrument was selected was that the mortgage price series was essentially the same as for the Treasury bond and GNMA CDR futures contracts.

mortgages are generally not available.[15] Losey found that, for twelve of the thirteen quarterly hedge positions taken, the changes in yields on mortgages and GNMA CDR futures were in the same direction; however, the futures yield movement ranged from 34 to 268 percent of the cash market movement. Thus, on a yield basis, hedging of mortgages by GNMA CDR futures was found to provide a less than perfect hedge.

Kolb, Corgel and Chiang (1982) developed a risk minimizing hedging strategy for mortgage loans based on duration analysis. The procedure used was based on the estimation of the price sensitivities of the hedged and hedging instruments. The authors argued that mortgage market participants are not always able to hedge an instrument with exactly the same characteristics as an instrument for which futures contracts are traded. This is because the risk, maturity, and coupon structures of the hedged and hedging instruments are generally mismatched. Because of this mismatch, the optimal hedge ratio for risk minimization will depend on the sensitivities of the prices of the instruments to changes in the risk-free rate of interest.

The optimal hedge ratio N is the number of futures contracts, j, used to hedge one unit of asset i, with the goal that, over the life of the hedge:

$$\Delta P_i + \Delta P_j(N) = 0, \qquad (3\text{-}34)$$

where P_i and P_j are the respective values of the instrument to be hedged

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[15]The loan yield used in the analysis was the average contract rate on conventional mortgage loans originated by all major lenders for the purchase of previously occupied homes. For details of the data used in this study, see Table 1, page eleven, of this article.

and the futures contract used as the hedging instrument. The problem is then to select the number of futures contracts, N, to trade in order to offset differing interest rate sensitivities of i and j so that equation (3-34) holds. To find N, one must solve:

$$\frac{\partial P_i}{\partial R_F} + \frac{\partial P_j}{\partial R_F}(N) = 0, \tag{3-35}$$

where R_F is one plus the risk-free interest rate. Using MacCauley's definition of duration, the authors found the risk-minimizing hedge ratio:

$$N = -\frac{R_j P_i D_i}{R_i P_j D_j} \cdot \frac{R_i/\partial R_F}{R_j/\partial R_F}, \tag{3-36}$$

where D_i and D_j are the durations of instruments i and j, and R_i and R_j are one plus the yields of the respective instruments.[16]

Using monthly data for commercial mortgage commitments and GNMA CDR future contracts as the hedged and hedging instruments, the authors found that $\partial R_i/\partial R_j = .421$, and used this value in conjunction with the durations of the instruments in equation (3-36) in generating various examples of hedged positions. The authors argued that their price sensitivity hedging strategy will substantially reduce hedging errors as compared to the naive hedging strategy of employing a one-to-one hedge ratio.

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[16]MacCauley's measure of duration for asset k is:

$$D_k = \frac{\sum_{t=1}^{k} t \cdot C_{tk}/(R_k)^t}{\sum_{t=1}^{k} C_{tk}/(R_k)^t},$$

where C_{tk} is the cash flow of asset k in the t^{th} time period, and R_j is one plus the discount rate on the asset.

Merton, Scholes and Gladstein (1978 and 1982) used simulation techniques to estimate the returns and risk of alternative investment strategies with call and put options on corporate stocks. In the earlier article, fully covered call option writing and a buying strategy which combined the purchase of call options on corporate stocks and commercial paper were evaluated. In the latter article, the risk-return distributions from uncovered put-writing and a put purchase strategy were estimated using simulation methods. Although the authors did not evaluate strategies using options on Treasury bond futures or other futures contracts, their research is relevant to the present study in that the options strategies were evaluated over a period of varying market environments and demonstrate how options strategies can be used to modify the risk-return patterns on the underlying instrument. In both analyses, the authors found that the return distributions were sensitive to the choice of the simulation period, the length of the holding period, and the choice of the option premium levels.[17]

Bookstaber (1982) employed the portfolio approach of Johnson, Stein, and Ederington in a simulation analysis applied to the hedging of mortgage loan commitments with Treasury bond futures and options on Treasury bonds. In this study, alternative hedge ratios were used in the simulations in order to derive the risk-return characteristics of different hedging strategies employing these instruments. Bookstaber estimated the risk-return

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[17]Merton, Scholes and Gladstein, in their 1982 article, discuss the sensitivity of the return to the simulation period, length of holding period, and the options premium on page three.

distributions for various combinations of futures and options used to hedge a mortgage bank's portfolio. The alternative hedging positions evaluated were short sales of Treasury bond futures, writing call options on Treasury bonds, buying put options on Treasury bonds, and a combined strategy of writing calls and buying puts on the Treasury bonds. The present study differs from Bookstaber's work in a number of ways. First, mortgage loan positions are being hedged rather than loan commitments. Second, Bookstaber considered only options on cash Treasury bonds, not options on Treasury bond futures. Additionally, a number of alternative hedge ratios were evaluated rather than using the risk-minimizing hedge ratio developed by Ederington.

Draper and Hoag (1983) used simulation techniques to generate the risk-return distributions of alternative hedging strategies involving cash holdings of Treasury bonds and Treasury bond futures and/or options on these futures instruments. They analyzed five alternative strategies for bond hedging consisting of the following positions: long bonds, long futures contracts, long call options on the bond futures, long bonds and short futures, and long calls and short bond futures. The strategies employing futures contracts used four alternative maturity dates for the contracts, and three alternative futures options' exercise prices were assumed in the strategies using options positions. For each strategy, it was assumed that the hedge was maintained for six months. For each future and/or futures options position taken, a one-to-one hedge ratio was constructed; i.e., there was no estimation of an optimal hedge ratio for the strategy employed.

Both the prices of the Treasury bond futures and the options on the futures were simulated in the analysis. Only call options prices were estimated in the analysis, and the options pricing model used in the study was the Black model developed previously in this chapter. Because options on Treasury bond futures were not yet traded at the time of their research, Draper and Hoag did not test the simulated model prices against actual trading prices of the options.

The authors found that, in a period of declining cash bond prices, both futures and options on these futures improved bond portfolio returns and reduced the standard deviation of the return in almost all of the cases. This research, thus, provides evidence that hedging strategies using futures options can improve the return and reduce the risk of a cash position during a period of declining cash instrument prices.

CHAPTER IV.

TESTS OF THE OPTIONS PRICING MODEL

In order to estimate the returns to strategies which incorporate the use of options on Treasury bond futures contracts, options prices must be approximated for periods when the contracts were not traded. The option pricing model used to approximate contract prices must provide reasonably close estimates of the prices at which the contracts would have traded. The purpose of this chapter is to test the predictive power of the Black option pricing model developed in Chapter III for pricing call and put options on Treasury bond futures contracts.

The data used in testing the Black model are described in the first section. There are only two parameters of the model which are not directly observable, the risk-free rate of interest and the variance of the fractional change in the futures price, or σ^2. Alternative procedures for estimating the variance are discussed in the second section, and the rationale for the selection of the procedure used is also presented. In the last section, the actual prices of traded call and put prices are compared to the Black model's estimates of these prices in order to measure the accuracy of the model.

Data Specifications

There are five parameters of the Black options pricing model: the price of the underlying futures contract, the exercise price of the option, the option's time to expiration, the risk-free rate of interest, and the variance rate of the fractional change in the price of the futures contract. The futures price which is used in testing the Black model is the first closing price of the Treasury bond futures contract on which the option is written on the date at which the option's price is estimated. A number of alternative exercise prices for both call and put options are used for each specified futures contract. These coincide with the actual exercise prices of traded options on the specified contracts on the specified trading dates.

The times to expiration for the options are derived from the Chicago Board of Trade's calendar of contract expiration dates. The variable used in the Black model is the fraction of a year until the option's expiration. The time to expiration is then the number of days until expiration divided by three hundred and sixty-five.

The proxy used for the risk-free rate of interest in the model is the market rate on Treasury bills with maturity closest to the expiration date of the option's contract. The variance rate that should be used is more problematic than the other model parameters. The criteria for selecting the variance rate are presented in the following section.

Futures Price Variance Estimation Techniques

The variance of the price of the underlying futures contract, or its volatility, is the most difficult of the Black model's parameters to estimate. The method of estimation used in this study incorporates the use of historical data in approximating the instantaneous variance.[1] Alternative specifications of the length of past observations are used to

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[1]Bookstaber (1981) provides an extensive description of alternative procedures for calculating the volatility of corporate stock prices and an analysis of the strengths and weaknesses of these procedures.

approximate the variance. Two alternative methods are then used to select the measure of volatility which is included in the options pricing model.

The historical variance estimate used as a proxy for the instantaneous variance is calculated by converting the observations on the futures price into rates of return where the rate of return is defined as:

$$R_t = F_t/F_{t-1}, \tag{4-1}$$

and F_t and F_{t-1} are the futures contract's prices at times t and t-1. The natural logarithms of R_t are then used to approximate the continuously compounded return. The mean of the rate of return is then calculated as:

$$m = 1/n \sum_{t=1}^{n} \ln R_t, \tag{4-2}$$

where n is the number of observations on R_t. The variance of the continuously compounded return is calculated as:

$$\sigma^2 = 365 \cdot \frac{1}{n} \sum_{t=1} (\ln R_t - m)^2. \tag{4-3}$$

Because daily observations for the futures price series are used, the sum of the squared deviations from the mean return is multiplied by three hundred and sixty-five in order to get an annualized rate of return.[2]

[2]Parkinson (1983) derived an alternative measurement procedure for estimating the variance of the rate of return based on the extreme values of the observed futures prices. This procedure uses the high and low prices of the futures contract for each unit time interval. He argued that this procedure provides a superior estimate to the use of the price levels for each unit time interval because the variance of the extreme value estimator is smaller for a given sample size. The estimates of the variance using Parkinson's procedure did not prove to be as good as the one month historical estimates using either the implied variance test or the pricing error tests described below.

Three alternative lengths of periods are used in calculating the historical variance -- one, three, and six months of daily observations of the futures contract prices. Two procedures are used to select the historical variance estimator to be used in the pricing model. The first procedure is a grid search method which is used to solve for the implied variance for a given option's price. In this procedure, the observable pricing model parameters are combined with a starting value of the unobservable variance to generate an implied price of the option. By using different values of σ^2, the Black model is solved iteratively until a value of σ^2 is found which minimizes the difference between the observed and estimated options prices. The values used for the implied standard deviation range from .05 to .40 in increments of .001 units. The value of σ^2 which minimizes the difference between the observed option price and the value generated by the Black model is then the implied variance of the option.[3]

The values of the historical variances for the alternative lengths of observations are then compared to the model's implied variance in order to determine which estimate most closely approximates the model's variance. The grid search procedure generated values of the implied standard deviations for both call and put options and alternative trading days, futures contracts and exercise prices are presented in Table 4-1, columns four and five. La Tané and Rendleman reason that, if the market is pricing options and risk efficiently, given the risk-free rate of interest, the same

[3]Belongia and Gregory (1984) used the same procedure to solve for the implied variance in order to test the efficiency of the Treasury bond futures options market.

Table 4-1. *Standard deviations of the return on futures contracts*

Date	Futures contract	Implied standard deviation: Exercise price	Implied standard deviation: Call option	Implied standard deviation: Put option[a]	Historical standard deviation: One-month	Historical standard deviation: Three-month	Historical standard deviation: Six-month
10/15/82	Mar 1983	70	.182	.223	.268	.223	.190
		72	.244	---			
		74	.195	---			
		76	.207	.190			
		78	.204	---			
01/14/83	Mar 1983	74	.149	.143	.137	.175	.202
		76	.219	.130			
		78	.126	.125			
		80	.137	.138			
04/15/83	Sep 1983	70	.140	.117	.110	.134	.148
		74	.113	.109			
		76	.104	.110			
		78	.107	.110			
		80	.107	---			
01/26/83	Mar 1983	68	.050	.116	.116	.166	.201
		70	.050	.147			
		72	.125	.143			
		74	.127	.131			
01/26/83	Jun 1983	68	.112	.116	.117	.162	.196
		70	.126	.133			
		72	.124	.131			
		74	.128	.135			
		76	.131	.116			
		78	.115	.131			

[a]Dashed line (---) means that no option price was reported for the contract on this date.

variance should apply to all options traded on a given futures contract on a particular day.[4] It is obvious from Table 4-1 that, for either calls or puts on a given contract and date, the implied variances for different exercise prices are not all equal.

For five of the six contracts for which implied standard deviations are calculated, the one-month historical variance estimates are substantially closer to the implied standard deviations than are either the three- or six-month estimates. This is true for each alternative exercise price used. Also, except for the options traded on October 15, 1982, the one-month historical estimates were in each case less than either the three- or six-month estimates.

In all cases except for contracts traded on October 15, 1982, the three- and six-month estimates were greater than any of the implied values. It is quite possible that the implied standard deviations for this date are an aberation, because options on Treasury bond futures contracts only began trading in this month and, as a consequence, the market might not have developed sufficiently to price the options efficiently.

A direct test of the efficacy of the alternative historical variances in estimating the variance of the futures return can also be used. In this test, the three alternative estimates are calculated for a sample of trading dates and these values are then combined with the other model parameter values to determine the estimated option prices. The estimated values of both call and put options for different contract months and

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[4]Belongia and Gregory (1984, p. 10) found that, judgmentally, the estimated differences of the implied variances were small. In half of the cases examined, the spread is .026 points or less.

exercise prices are then compared to the actual prices for the options traded on these contracts. The resulting estimates using the alternative standard deviations are presented in Table 4-2 for call options and Table 4-3 for put options together with the observed option prices. The historical variance estimates and the other model parameters used are given in Table A-1 of the appendix.

The results of the direct test of the predictive power of the Black model using the alternative historical variances are consistent with the grid search procedure. In approximately eighty percent of the cases, using the one-month variance estimate results in a more accurate estimate of the call option price than when the three-month estimate is used. In no case does the six-month variance estimate generate the best estimate of the call option price. The results of the test are the same with respect to the accuracy of the put price estimates. In more than two-thirds of the cases presented in Table 4-3, the smallest absolute prediction error results from using the one-month variance estimate, and in none of the cases is the absolute prediction error the smallest when the six-month variance estimate is used.

The estimates of the call and put option prices using the one-month variance estimates do not exhibit a systematic bias. Of the fifty-five alternative call price estimates, in twenty-four cases the estimated price is larger than the actual price, while, in thirty-one, it is less. The same result holds for the put price estimates using the one-month estimates. In twenty-nine of the fifty-one cases, the estimated option price is greater than the actual price, and, for twenty-two of the cases, it is

Table 4-2. Actual and estimated call options prices

Date	Contract	Exercise price	Actual	Estimated variance One-month	Three-month	Six-month
11/15/82	Mar 1983	68	9.000	8.753	9.090	9.090
		70	7.297	7.082	7.530	7.531
		72	5.828	5.566	6.117	6.119
		74	4.469	4.239	4.870	4.871
		76	3.391	3.123	3.796	3.797
		78	2.437	2.222	2.896	2.896
		80	1.703	1.527	2.163	2.165
01/14/83	Mar 1983	68	8.844	8.810	8.016	8.001
		70	6.844	6.837	6.895	6.952
		72	4.875	4.917	5.051	5.182
		74	3.234	3.164	3.418	3.621
		76	1.672	1.751	2.096	2.347
		78	.703	.894	1.148	1.400
		80	.297	.299	.556	.763
01/14/83	Jun 1983	70	6.437	6.424	6.771	7.085
		72	5.000	4.879	5.327	5.706
		74	3.703	3.547	4.072	4.497
		76	2.672	2.460	3.019	2.611
		78	1.703	1.623	2.170	1.924
		80	1.031	1.017	1.215	1.133
01/26/83	Jun 1983	68	5.187	5.219	5.658	6.057
		70	3.781	3.672	4.258	4.736
		72	2.516	2.402	3.081	3.606
		74	1.625	1.449	2.138	2.672
		76	1.000	.802	1.422	1.926
		78	.625	.405	.906	1.350
02/15/83	Jun 1983	68	5.625	5.454	5.602	5.982
		70	4.000	3.906	4.110	4.593
		72	2.672	2.620	2.863	3.412
		74	1.641	1.635	1.886	2.448
		76	.891	.946	1.172	1.696
		78	.500	.505	.686	1.134
		80	.281	.249	.378	.732

Table 4-2. *Continued*

Date	Contract	Exercise price	Actual	Estimated variance One-month	Three-month	Six-month
03/15/83	Jun 1983	68	8.062	7.996	7.997	8.120
		70	6.062	6.153	6.146	6.376
		72	4.234	4.465	4.448	4.801
		74	2.703	3.019	2.994	3.450
		76	1.516	1.883	1.853	2.355
		78	.734	1.075	1.047	1.523
		80	.359	.559	.537	.931
03/15/83	Sep 1983	74	3.047	3.192	3.391	4.093
		76	2.047	2.224	2.415	3.137
		78	1.375	1.357	1.656	2.352
		80	.781	.680	1.092	1.725
		82	.453	.366	.693	1.238
04/14/83	Sep 1983	72	6.156	5.978	6.201	6.355
		74	4.406	4.357	4.679	4.882
		76	2.891	2.997	3.375	3.615
		78	1.844	1.892	2.320	2.575
		80	1.078	1.111	1.516	1.760
		82	.594	.601	.940	1.154
06/15/83	Dec 1983	72	3.844	4.008	3.982	4.207
		74	2.625	2.799	2.768	3.025
		76	1.609	1.815	1.820	2.082
		78	.984	1.156	1.128	1.369
		80	.563	.682	.658	.860

Table 4-3. Actual and estimated put options prices

Date	Contract	Exercise price	Actual	Estimated variance		
				One-month	Three-month	Six-month
11/15/82	Mar 1983	68	.469	.326	.663	.663
		70	.781	.609	1.057	1.058
		72	1.156	1.047	1.598	1.600
		74	1.859	1.674	2.304	2.306
		76	2.625	2.512	3.185	3.186
		78	3.750	3.566	4.240	4.241
		80	4.969	4.824	5.461	5.462
01/14/83	Mar 1983	70	.031	.014	.064	.131
		72	.094	.079	.214	.345
		74	.344	.311	.565	.768
		76	.813	.882	1.228	1.479
		78	1.813	1.921	2.264	2.516
		80	3.406	3.401	3.657	3.864
01/14/83	Jun 1983	70	.453	.439	.785	1.100
		72	.781	.837	1.286	1.665
		74	1.406	1.450	1.975	2.400
		76	2.313	2.308	2.867	3.314
		78	3.328	3.415	3.963	4.404
		80	4.797	4.754	5.924	5.661
01/26/83	Jun 1983	68	.313	.322	.761	1.160
		70	.922	.723	1.308	1.786
		72	1.625	1.399	2.078	2.603
		74	2.687	2.393	3.083	3.616
		76	3.906	3.693	4.314	4.817
		78	5.406	5.244	5.744	6.189
02/15/83	Jun 1983	68	.297	.352	.500	.880
		70	.672	.759	.964	1.446
		72	1.328	1.428	1.672	2.220
		74	2.281	2.399	2.650	3.212
		76	3.500	3.665	3.891	4.415
		78	4.953	5.180	5.360	5.808
		80	6.937	6.879	7.008	7.362

Table 4-3. *Continued*

03/15/83	Jun 1983	68	.063	.062	.056	.185
		70	.125	.187	.175	.410
		72	.313	.467	.447	.803
		74	.719	.989	.962	1.420
		76	1.469	1.821	1.792	2.293
		78	2.687	2.981	2.955	3.430
		80	4.281	4.434	4.415	6.382
03/15/83	Sep 1983	72	1.000	1.369	1.279	1.916
		74	1.719	2.108	2.007	2.710
		76	2.703	3.061	2.957	3.679
		78	3.937	4.222	4.123	4.819
04/14/83	Sep 1983	74	.281	.567	.888	1.092
		76	1.141	1.128	1.526	1.765
		78	2.000	1.983	2.411	2.665
		80	---	3.143	3.547	3.791
06/15/83	Dec 1983	72	.844	1.000	.979	1.204
		74	1.531	1.718	1.687	1.945
		76	2.578	2.692	2.660	2.923
		78	3.781	3.919	3.890	4.132
		80	5.281	5.366	5.342	5.544

less. This is not true for the estimated call or put option prices when either the three- or six-month variance estimates are used. In all but two cases, the estimated call option prices are greater than the observed prices using the three-month estimate, and in only one case is the estimated put price less than the observed price when these variance estimates are used.

The Predictive Power of the Black Options Pricing Model

In the last section, it was demonstrated that the estimated options prices generated from the Black model most closely approximate the observed prices of call and put options when the one-month historical variance estimates are used in the model. In order to test the usefulness of the Black model using this variance estimate, the values of the estimated call and put prices must be compared to the observed option prices.

Because trading in Treasury bond futures options began in October, 1982, estimated options prices generated from the Black model on futures contracts with maturities in 1983 are used to test the model's accuracy. Estimates for these futures contracts for alternative dates and exercise prices are presented in Tables 4-2 and 4-3 and Table A-2 of the Appendix. A total of eighty-four call prices and seventy-nine put prices are estimated.

There is no evidence that the estimates of call options prices generated from the Black model are biased estimates of actual call prices. Of the eighty-three prices estimated, thirty-eight were less than the actual prices and forty-five are greater than the actual prices. The call estimates are also reasonably close estimates of the actual prices. Fifty-nine of these differ by less than two-tenths of a point, or $200, from the actual prices of the traded options, while, in twenty-five cases, the difference is larger than this amount. If the 1982 trading date contracts are excluded, only eleven of the sixty-five estimates differ from the traded prices by more than two-tenths of a point, while fourteen of the nineteen 1982 differences are greater than this.

A probable explanation for the greater predictive power of the model for the 1983 trading dates is the newness of the market. This possibly resulted in inefficiencies which were corrected as the market developed greater breadth and traders of the contract became more experienced in pricing these contracts. The closeness of the estimated and actual call prices is also demonstrated by the fact that twenty-eight of the estimated prices were within one-tenth of a point, or $100, of the traded prices. The mean square error of the eighty-four estimates is .0423, and the root mean square error is .2063, or approximately $200.

As is the case for the estimated call option prices, the put price estimates generated from the Black model and the put-call parity equation are reasonably close estimates of the actual put trading prices. There is little evidence of a systematic bias in the put price estimates in either direction. Forty-five of the estimated put prices are greater than the actual trading prices, while thirty-four are less.

In fifty-eight of the seventy-nine cases, the estimated price is within $200 of the actual price, while the difference is greater than this in the other twenty-one cases. In thirty-two of the cases, the difference between the estimated and actual prices is less than one-tenth of a point; i.e., in forty percent of the cases, the model's estimates are within $100 of the actual prices. The mean square error and root mean square error of the seventy-nine observations are .0569 and .2385, respectively.

As is the case for call options, a greater proportion of the estimated put prices for the 1982 trading dates are substantially different from the actual prices than is true for the 1983 trading date estimates. Seven of the eighteen estimates for 1982 dates differ by more than two-tenths of a point from the actual prices. When the put-call parity equation is used to price put options, a substantial estimation error for a call price will generally also cause a substantial error in the estimated put price.

There is no evidence that the accuracy of the estimated option price is correlated with either the time to expiration of the option or the difference between the futures contract's price and the exercise price of the option. For example, the estimated put options prices are substantially worse for the September, 1983, contract estimated for March 15, 1983, than the June 15, 1983, estimates of the December, 1983, option prices, even though the times to expiration are almost exactly the same. In some cases, the farther in-the-money are the contracts, the closer are the estimated and actual prices, while, in other cases, the estimated prices of these options have a greater magnitude of error than do the out-of-the-money options.

These tests of the Black options pricing model using the one-month historical variance estimate provide evidence that the model is appropriate for estimating options prices for those periods when actual trading prices do not exist. It was demonstrated that the model generates unbiased estimates for both call and put options, and that these estimates are within acceptable ranges of the actual option prices.

CHAPTER V. EMPIRICAL TESTS OF THE HEDGING STRATEGIES

In this chapter, the results of the empirical tests of the alternative hedging strategies discussed in Chapter III are presented. In the first section, the specifications of the data used in the tests are described. The procedure used in estimating the risk-minimizing hedge ratio and the results of the estimation are presented in the second section. The first part of the third section contains a discussion of the alternative strategies evaluated and also the procedures used in estimating the distribution of returns for the strategies. The results of the empirical tests are then presented and analyzed. The fourth section deals with the estimation of the return distributions to the strategies using a simulation technique of reversing the time series of observations on the mortgage and futures contract prices. The results of the estimates for the two estimations are compared and conclusions about the efficacy of the alternative strategies are presented in the last section.

Data Specifications

In this section, the structure of the data used in the empirical tests of the strategies and their sources is described. The first difficulty in evaluating the strategies is to construct an appropriate price series for fixed-interest-rate mortgages because there is no published series of market prices of mortgages. In order to construct a mortgage price series, a market mortgage interest rate series must be used. The series selected for this analysis was the average contract commitment rate for fixed-interest, seventy-five percent loan-to-price ratio, twenty-five year conventional mortgages. This monthly series is published in the *Federal Home Loan Bank Board Journal*. The commitment rate was used rather than the closing rate on mortgage loans because it is likely to be a better measure of the current market rates on mortgages and, thus, effective rates in the secondary market.

This mortgage interest rate series was converted into a mortgage price series by using a table of "Prepayment Mortgage Values" published in the *Thorndike Encyclopedia of Banking and Financial Tables, Revised Edition* (1980). The table used assumed an eight percent twenty-five year mortgage which will be prepaid in the twelfth year. It was, thus, assumed that twenty-five year mortgages with an interest rate of eight percent, a loan-to-price ratio of seventy-five percent and expected prepayment in the twelfth year are being hedged. An eight percent mortgage price basis was selected because Treasury bond futures contracts are based on an eight percent coupon Treasury bond, and this puts the mortgage and Treasury bond futures price series on the same basis.

The first closing prices of the Treasury bond futures contracts were used in the hedges for both the dates of initiation and lifting of the hedges, respectively. Since it was assumed that each hedge was initiated on the fifteenth day of the month and maintained for six months, the first closing prices of the futures contracts on these dates was used. If the fifteenth day was not a trading day, the date closest to this day was used. The futures price data were taken from the Chicago Board of Trade Foundation commodity price data base tapes for the years 1977 through 1983.

The futures price series used in calculating the historical variance estimates were the daily first closing prices of the Treasury bond futures contracts used for each hedging period. The data used in calculating the returns from placing stop orders were also daily observations on the Treasury bond futures contracts' prices for the selected hedging periods. Since the stop orders are executed if the price of the futures contract exceeds a specified level at any time, daily highs of the quoted prices were used. In all cases, the futures price data were taken from the Board of Trade's data tapes.

The structure of the data used in the Black options pricing model and its sources were discussed in Chapter IV. In this section, the specific type of data required for the simulations of the options prices and its sources are described. Because the hedging periods used in the analysis cover only the period for which the underlying futures contracts were traded, the futures prices used in estimating the option prices were obtained from the data tapes. Since call and put options' exercise prices are established at even-numbered integers in increments of two points, the exercise prices used in the simulations were determined on this basis. For each hedging period, both call and put options prices were estimated for exercise prices closest to the initial futures price and meeting the above specifications, and for exercise prices two points above and below this value.

The futures contract used for each hedging period was the contract nearest to maturity for which options on the contract had not expired at the time the hedge was lifted. Consequently, because the option contracts initially have from six to eight months until expiration, the interest rate used as a proxy for the risk-free rate was the average yield on six-month U.S. Treasury bills traded in the secondary market during the week containing the fifteenth day of the month. This series was constructed from data published in the *Federal Reserve Bulletin*.

Because Treasury bond futures options were not traded for most of the period being analyzed, the time to expiration must be approximated. This was done by using the dates of expiration of the underlying futures contracts and the Board of Trade's specifications for expiration dates of options on Treasury bond futures discussed in Chapter II. The data specifications for calculated values of the historical variances estimated were described in Chapter III.

Estimation of the Risk-Minimizing Hedge Ratio

In Chapter III, it was demonstrated that the ratio of futures contracts to cash instruments which minimizes the variance of the return to a hedged position can be estimated by the coefficient of the regression of the change in the cash instrument's price on the change in the price of the futures contract. If the purpose of the hedge is to minimize the variance of the return, or the risk, of a cash position, this ratio can be considered to be the optimal hedge ratio. The results of estimations of this ratio using alternative time periods and cash/futures relationships are presented in this section. These estimates are then used in determining the risk-return distributions of the alternative hedging strategies.

It was assumed that each position was maintained for six months. Consequently, the data used in the estimations were monthly observations for the six-month changes in the mortgage and Treasury bond futures contract prices. The first hedge was assumed to be established on July 15, 1978, and the last hedge lifted on December 15, 1983.

The slope coefficient for the regression of the changes in mortgage prices on changes in the futures contract prices using the six-month periods was .7040. The coefficient was significant at the .0001 level. In order to minimize the risk of the position, each $100,000 of mortgage holdings would be hedged by selling .704 Treasury bond futures contracts. The results of the alternative estimations are presented in Table 5-1.

Table 5-1. Regression estimates for changes in mortgage and futures prices

	Uncorrected		Corrected for autocorrelation	
	Contemporary	Lagged	Contemporary	Lagged
	Futures price changes		Futures prices	Futures prices
Constant	.0732	.4587	-.6379	.2230
	(.115)[a]	(1.141)	(-.768)	(.444)
Futures prices	.7040	.8551	.4596	.7648
	(8.6713)	(16.787)	(4.534)	(11.729)
n	60	59	60	59
R^2	.5645	.8318	.2999	.7295
Durbin-Watson Statistic	.8648	1.1796		

[a]Values of the t-statistics are shown in the parentheses.

The coefficient of determination, R^2, was .5645, which would indicate that a substantial proportion of the movement in mortgage prices is not offset by movements in the futures prices. As was discussed in Chapter III, the coefficient of determination is an estimate of the efficiency of the hedge, or the percentage reduction in the variance of the hedged position as compared to the unhedged position.

The value of the Durbin-Watson statistic indicates that there was first order autocorrelation of the residuals. When the estimates were corrected for autocorrelation, the optimal hedge ratio estimate declined substantially to .4596 and the coefficient of determination was reduced to .2999.[1] The coefficient for changes in futures prices was still significant at the .0001 level.

The values of R^2 for both the uncorrected and corrected regressions indicate that a substantial part of the variation in the cash prices is not eliminated by the hedge. Hilliard and Haney (1982) estimated the lag structure of the relationship between long-term government bond yields and mortgage interest rates. They found that, during the latter part of the 1970s, changes in mortgage interest rates lagged behind changes in the yields on long-term government bonds by approximately one month. To the extent that yields on government bonds and Treasury bonds futures prices

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[1]The procedure used to correct for autocorrelation of the residuals in the PROC AUTOREG subroutine of the Statistical Analysis System package. The procedure is equivalent to a generalized least squares estimate of the regression coefficients with appropriate weights. The lag length specified for the procedure is six periods because the autocorrelation coefficient for this length is significantly different from zero.

move synchronously, there will also be approximately a one-month lag between changes in Treasury bond futures prices and mortgage prices.

When the optimal hedge ratio was reestimated incorporating the lagged relationship between changes in futures prices and mortgage prices, both the value of the hedge ratio and the coefficient of determination of the regression increased substantially. This was true for both the regressions not corrected for, and corrected for autocorrelation of the residuals. The respective uncorrected and corrected hedge ratios were .8551 and .7648; thus, each unit value of the cash position is hedged with a larger futures position using these estimates. The increase in the R^2 value from .2999 to .7295 for the corrected estimates indicates that using the lagged position would increase the efficiency of the hedge by approximately forty-three percent.

In essence, the incorporation of the lagged relationship between the futures and cash market positions creates an anticipatory hedging situation. Assume that a financial institution is planning to hedge a cash holding of mortgages at time t and the position will be maintained until time t+n. The futures transaction is made at time t-1 and lifted at t+n-1. Thus, to initiate the hedge, the futures market transaction must be undertaken in the period before the cash position is established and lifted one period prior to the end of the hedge.

If the cash position being hedged is newly issued mortgage loans, this strategy should not create substantial difficulties, since institutions which lend on these mortgages generally can predict within a narrow range what quantity of commitments will be taken down during the next month.

Also, if seasoned mortgages are being continuously hedged by rolling over the futures position, the difficulties of hedging the position would be substantially reduced, since only when the initial position is established will there be uncertainty in determining the size of the futures position to take.

The return distributions for the alternative hedging strategies using each of the hedge ratios and contemporaneous and lagged hedge position are presented in the following section.

Return Distributions for Alternative Strategies

The purpose of any hedging strategy is to alter the risk-return distribution from that of the unhedged position. In this section, the means and standard deviations of the returns for the alternative hedging strategies are analyzed for the contemporaneous and lagged mortgage and futures cases developed in the previous section.

For each strategy, it was assumed that the cash and futures positions were held for six months and that the futures contract used was the contract closest to maturity for which options had not yet expired at the time the hedge was lifted. A total of sixty hedge positions were established for the contemporaneous series and fifty-nine for the lagged series. For each of the alternative strategies, the mean and standard deviation of the returns were estimated for four alternative series -- the contemporaneous and lagged series using both the risk-minimizing hedge ratios uncorrected for autocorrelation, and corrected for autocorrelation.

As Merton, Scholes and Gladstein (1982, p. 3) have emphasized, "The return statistics for the various option strategies are sensitive to the choice of the simulation period and to the length of the holding period between revisions." Although their research was conducted on stock options, this is true for any study of alternative investment strategies based on past history. The returns to option strategies are also affected by the choice of premium levels for the options. Because the average returns to options positions are especially sensitive to the periods chosen, they provide unreliable indicators of expected future returns. The results presented in this section should be considered to be indicators of the returns for one sample, not a definitive test of the alternative strategies.

The means and standard deviations of the returns for the strategies evaluated for the alternative periods and hedge ratios are presented in Tables 5-2 and 5-3. The maximum and minimum returns over the periods are given in Table 5-6, and the returns for each period using the corrected hedge ratio are presented in Tables A-3 and A-4 of the appendix.

Although the sample period of July, 1978, through December, 1983, was one in which interest rates generally increased and mortgage and futures prices consequently declined, there was substantial variation in both the size of price change and the direction of change. For thirty-eight of the sixty periods, the mortgage price fell with a maximum decrease of 20.030 points. The largest increase in the mortgage price was 12.680 points for this period.

The mean return for the unhedged mortgage position over the period was -1.763 points (a $1,763 loss on a $100,000 position) and its standard deviation was 7.007 points. Both the loss to the position and the

Table 5-2. Mean and standard deviation of returns for alternative strategies: Contemporaneous mortgage and futures series (in thousands of dollars)

Strategy	Not corrected for autocorrelation		Corrected for autocorrelation	
	Mean return	Standard deviation of return	Mean return	Standard deviation of return
Unhedged mortgages	-1.763	7.007	-1.763	7.007
Mortgages and futures				
One-to-one hedge ratio	.845	5.126	.845	5.126
Risk-minimizing hedge ratio	.073	4.624	- .565	4.972
Mortgages, futures and call options[a]				
Exercise price > futures price	- .982	5.131	-1.253	5.497
Exercise price = futures price	-1.149	5.379	-1.363	5.710
Exercise price < futures price	-1.282	5.606	-1.449	5.917
Mortgages and put options				
Exercise price > futures price	- .889	5.132	-1.193	5.501
Exercise price = futures price	-1.150	5.384	-1.363	5.714
Exercise price < futures price	-1.398	5.664	-1.525	5.951
Mortgages, futures and stop orders				
Three point order	- .682	5.578	-1.057	5.690
Four point order	- .550	5.282	- .971	5.464
Five point order	- .423	4.915	- .863	5.168

[a]All stop order and options positions are estimated using the risk-minimizing hedge ratios of .7040 and .4569, respectively.

Table 5-3. Mean and standard deviation of returns for alternative strategies: Lagged mortgage and futures series (in thousands of dollars)

Strategies	Not corrected for autocorrelation: Mean return	Not corrected for autocorrelation: Standard deviation of return	Corrected for autocorrelation: Mean return	Corrected for autocorrelation: Standard deviation of return
Unhedged Mortgages	-1.728	7.061	-1.728	7.061
Mortgages and futures				
One-to-one hedge ratio	.829	3.095	.829	3.095
Risk-minimizing hedge ratio	.461	2.896	.227	2.975
Mortgages, futures and call options[a]				
Exercise price > futures price	- .813	3.815	- .910	3.972
Exercise price = futures price	-1.007	4.220	-1.083	4.373
Exercise price < futures price	-1.155	4.626	-1.216	4.777
Mortgages and put options				
Exercise price > futures price	- .699	3.824	- .808	3.982
Exercise price = futures price	-1.007	4.228	-1.084	4.381
Exercise price < futures price	-1.296	4.649	-1.342	4.794
Mortgages, futures and stop orders				
Three point order	- .474	4.918	- .606	4.878
Four point order	- .311	4.568	- .461	4.535
Five point order	- .141	4.284	- .377	4.231

[a]All stop order and options positions are estimated using the risk-minimizing hedge ratios of .7040 and .4569, respectively.

variability of the return were substantially reduced when the cash position was hedged using either a one-to-one hedge ratio or the risk-minimizing ratio. The mean return for the one-to-one ratio using the contemporaneous mortgage/futures strategy was .845 and its standard deviation was 5.126. If the cash position is hedged using the lagged strategy, the return for the period using the optimal hedge ratio was substantially closer to zero, .227, and its standard deviation was also considerably smaller, 2.975. Thus, hedging the cash position using the lagged futures strategy and the optimal hedge ratio corrected for autocorrelation of the residuals would have increased the mean return by approximately $1,950 for each $100,000 of mortgages held and reduced the variance of the return by eighty-two percent. Also, the minimum return was reduced by almost seventy-five percent for this strategy.

Selling futures contracts and simultaneously purchasing call options on these contracts results in a lower mean return and a larger variance of the return than when only futures are used in the hedge. This is the case, no matter whether the contemporaneous or lagged position was used, the hedge ratio selected, and the relationship between the initial futures price and the option's exercise price. For example, using the lagged futures and the corrected hedge ratio, the mean returns for options with exercise prices greater than, equal to, and less than the initial futures price were -.910, -1.083 and -1.216, respectively. The respective values of the standard deviations of returns were 3.972, 4.373, and 4.777.

Because the sample period was one of generally declining futures prices, the purchase of call options would not have been necessary, ex post, in the majority of cases. The purchase of call options was more beneficial in periods when the futures price was increasing, since the call options would not be exercised when the futures price at the end of the hedging period was less than the option's exercise price. The mean return in each case was also largest for the out-of-the-money call options and the standard deviation of the return the smallest. This is to be expected, because the farther out-of-the-money the call option, the lower its price will be; thus, the lower is the cost of purchasing an option that expires without being exercised. The use of call options on the futures reduced the maximum loss by from forty to fifty percent from that of the unhedged position, but was also greater than for using futures contracts without purchasing options.

The means and standard deviations for hedging by purchasing put options on the underlying futures contracts were essentially the same as for futures and call options strategies for equal options exercise prices. This is to be expected, because, as was demonstrated in Chapter III, the purchase of put options replicates the sale of futures contracts and the purchase of call options on the contracts if the options have the same exercise prices. For each of the exercise prices, the maximum and minimum returns were also essentially the same for the alternative strategies.

During a period of declining cash and futures prices, it would be better to hedge using call options with exercise prices greater than the selling price of the futures contracts in that the mean return was larger and the standard deviation of returns was smaller the larger was the exercise price relative to the future's selling price. This was also true for the put options hedging strategy.

When stop orders were placed on the futures contracts, the mean returns were smaller and the standard deviations larger than when the orders were not used. This was the case for all three of the levels of stop orders used, for both price series, and for both hedge ratios used. For the three alternative stop-out limits used, the closer was the stop-out price to the initial selling price of the futures contract, the smaller was the mean return and the greater was the standard deviation of returns.

The closer the stop-out price is to the initial selling price of the futures contract, the greater is the probability that the futures contract will be offset during the hedging period when, ex post, it should not have been. This was the case for the periods used in the sample. For the lagged hedging strategy, the stop out order was executed in thirty of the fifty-nine periods for a three point limit above the futures selling price, twenty-four times for the four point limit, and twenty times for the five point limit. For the three point limit in fourteen of these periods, the futures price at the end of the period was less than the initial selling price, while this was true for only nine periods for the four point limit and only seven periods for the five point limit. In almost fifty percent of the periods, the stop order was executed when, in retrospect, it should not be when the loss limit was three points; this was true for thirty-five percent of the periods for the five point limit.

The maximum loss for the sample period was from five to six points larger when the stop-out orders were used than when there were no limits established for the futures price. For each limit, this occurred in periods when the futures contract position was closed out and the mortgage and futures prices subsequently declined.

The results of the sample period estimates indicate that the pure hedging strategy of selling futures contracts outperformed each of the stop order and options positions in terms of either the mean return or the standard deviation of returns criteria. These results are consistent with what would be expected from the theory of hedging. The purpose of using either options or stop loss orders is to reduce the losses from declines in the cash instrument's price without sacrificing the gains from increases in its price. During periods of declining cash prices, as was the case for the sample period, the futures-only strategy should outperform the other strategies.

Using the mean-variance criterion, it is not possible to determine whether stop order strategies are better than options strategies. Although the mean return was larger for all of the stop order positions than for any of the call or put options strategies, in some cases, the standard deviation was also larger for the stop order positions than for the options strategies. For example, the five point stop order had the largest mean return and smallest variance of the stop order strategies, while the same was true for the options strategies with exercise prices greater than the initial futures price. However, although the mean return was larger for this stop order limit than for the options position, its standard deviation of the return was also larger. This was also true for the stop order and options strategies with the smallest mean returns. The choice of using stop orders or options positions would then depend on the utility functions of the individual hedger.

If, however, the purpose of the hedge is to avoid large losses while not sacrificing the opportunity for large gains, the options strategies in this case should be used. The maximum and minimum returns were larger for all of the options positions than for any of the stop order positions.

Returns to the Hedging Strategies for the Reverse Series

The advantage of using options on futures contracts in the hedging strategy is that the purchase of an options contract gives the holder the right, but does not obligate him/her, to take a position in the futures market. Consequently, an options strategy should provide a relatively better return to the hedged position during a period of generally rising prices for the cash and futures instruments. Placing stop orders on the futures contracts should also increase the returns to the pure futures hedge since the futures position would be closed out if its price increased by a sufficient amount.

In order to evaluate the risk-return distributions of the alternative strategies being analyzed, a simulation of the mortgage, futures and options prices was undertaken. The simulation procedure used was to reverse the mortgage and Treasury bond futures price series. The prices of the mortgages and futures contracts at the time the hedge was placed for the initial price series then became the end-of-period prices, and the prices at the termination of the hedges became the initial instrument prices.

The values used in the option pricing model to estimate the call and put prices were then the observed values at the end of each hedging period. For example, the six-month Treasury bill rate for January 15, 1983, was proxy for the risk-free of interest for the hedge for the period from January 15, 1982, through July 15, 1982. The historical variance used in the hedge was estimated from the date of the end of the original hedge using daily futures prices for the next month.

As in the case for the initial estimates, the risk-return distributions for the alternative strategies were estimated for the contemporaneous mortgage and futures series and the lagged series using both the corrected and uncorrected risk-minimizing hedge ratios. The results of the estimations of the mean return and the standard deviations of the returns are presented in Tables 5-4 and 5-5, the maximum and minimum returns Table 5-6 and the returns for each period in Tables A-5 and A-6 of the Appendix.

The mean returns for the unhedged position and for the hedged positions using only futures contracts were simply the values for the initial series, but with the opposite signs. The standard deviations of the returns to these positions were the same for both the initial and reverse price series.

Using the lagged relationship of mortgage and futures prices and the corrected optimal hedge ratio, the standard deviations of the returns were smaller for all of the strategies evaluated, but the mean returns were also less than for the unhedged position in every case. Thus, hedging using different combinations of futures, stop orders, and options would have reduced the variance of the unhedged position, but only by sacrificing a larger return. By reducing the ratio of futures contracts sold per unit of mortgages held, the returns could have been increased, as is evident by

Table 5-4. Mean and standard deviation of returns for alternative strategies: reverse price series and contemporaneous mortgage and futures prices (in thousands of dollars)

Strategy	Not corrected for autocorrelation		Corrected for autocorrelation	
	Mean return	Standard deviation of return	Mean return	Standard deviation of return
Unhedged mortgages	1.763	7.007	1.763	7.007
Mortgages and futures				
One-to-one hedge ratio	-.845	5.126	0.845	5.126
Risk-minimizing hedge ratio	-.073	4.624	.565	4.972
Mortgages, futures and call options[a]				
Exercise price > futures price	.064	5.190	.654	5.682
Exercise price = futures price	.400	5.522	.874	5.932
Exercise price < futures price	.754	5.842	1.105	6.171
Mortgages and put options				
Exercise price > futures price	.183	5.194	.732	5.679
Exercise price = futures price	.403	5.530	.875	5.937
Exercise price < futures price	.661	5.840	1.044	6.169
Mortgages, futures and stop orders				
Three point order	.652	6.005	1.038	6.214
Four point order	.572	5.627	.986	5.901
Five point order	.483	5.189	.896	5.593

[a]All stop order and options positions are estimated using the risk-minimizing hedge ratios of .8551 and .7648, respectively.

Table 5-5. Mean and standard deviation of returns for alternative strategies: Reverse series and lagged mortgage and futures price series (in thousands of dollars)

Strategies	Not corrected for autocorrelation: Mean return	Not corrected for autocorrelation: Standard deviation of return	Corrected autocorrelation: Mean return	Corrected autocorrelation: Standard deviation of return
Unhedged mortgages	1.728	7.061	1.728	7.061
Mortgages and futures				
One-to-one hedge ratio	-.829	3.095	-.829	3.095
Risk-minimizing hedge ratio	-.461	2.896	-.227	2.975
Mortgages, futures and call options[a]				
Exercise price > futures price	-.320	4.589	-.101	4.759
Exercise price = futures price	.076	5.073	.252	5.215
Exercise price < futures price	.498	5.154	.649	5.330
Mortgages and put options				
Exercise price > futures price	-.175	4.599	.027	4.764
Exercise price = futures price	.078	5.083	.254	5.223
Exercise price < futures price	.384	5.095	.547	5.295
Mortgages, futures and stop orders				
Three point order	.398	5.760	.541	6.030
Four point order	.320	5.275	.464	5.648
Five point order	.226	4.751	.384	4.816

[a]All stop order and options positions are estimated using the risk-minimizing hedge ratios of .8551 and .7648, respectively.

Table 5-6. Maximum and minimum returns for alternative strategies: Corrected hedge ratio (in thousands of dollars)

	Initial series: Contemporaneous		Initial series: Lagged		Reverse series: Contemporaneous		Reverse series: Lagged	
Strategy	Maximum	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum	Minimum
Unhedged mortgages	12.680	-20.030	12.680	-20.030	20.030	-12.680	20.030	-12.680
Mortgages and futures								
One-to-one hedge ratio	16.120	-11.217	9.322	- 4.814	11.217	-16.120	4.814	- 9.322
Optimal hedge ratio	10.455	-15.980	9.484	- 5.400	15.980	-10.455	5.400	- 9.484
Mortgages, futures and call options								
Exercise price > futures price	10.524	-17.006	10.197	- 8.774	16.922	-12.644	13.965	-13.035
Exercise price = futures price	11.093	-17.393	11.083	- 9.230	17.483	-12.454	14.938	-13.665
Exercise price < futures price	11.579	-17.866	11.797	- 8.540	17.998	-11.959	15.849	-13.349
Mortgages and put options								
Exercise price > futures price	10.582	-16.955	10.278	- 8.626	17.010	-12.611	14.203	-12.939
Exercise price = futures price	11.106	-17.407	11.049	- 9.277	17.497	-12.464	15.014	-13.665
Exercise price < futures price	11.548	-17.947	11.647	- 8.674	17.938	-12.032	15.763	-13.463
Mortgages, futures and stop orders								
Three point order	10.899	-15.980	9.836	-11.501	17.876	-14.131	16.125	-14.564
Four point order	10.827	-15.980	8.975	-10.678	17.876	-14.605	16.125	-14.524
Five point order	10.328	-15.980	7.565	-11.443	17.387	-12.638	16.125	-13.930

comparing the returns to the alternative strategies for the uncorrected and corrected hedge ratios, the uncorrected ratio being the larger one.

The mean return for each of the alternative options and stop order positions was larger than when only futures contracts were used. This supports the argument for the use of these strategies; that, during a period of increasing prices, the returns from their use will be greater than for hedges using only futures contracts. However, the variance for each alternative option and stop order strategy was also greater than for the futures-only hedge.

The results of the futures/call options and put options strategies support the evidence from the initial series that buying put options replicates selling futures and buying call options with the same exercise prices as the puts.

Although the variance of the returns was less the larger was the option's exercise price, as was the case for the initial series, the mean return was smaller the higher the option's exercise price. The estimated difference in the mean return was from -$520 to -$750 for using options with $4,000 higher exercise prices.

The mean-variance distributions of the stop order strategies were also altered when the reverse price series was used. The closer the stop order was placed to the initial futures price, the larger was the mean return and the standard deviation of the returns. In a period over which the futures contract price generally increases, the probability that the stop order is executed when, ex post, it should not have been is reduced. Even when the stop order was only three points above the initial futures selling price, there were only five periods for which the stop order was executed and the futures price subsequently fell below its initial price. Although the mean return was larger the closer the stop order to the original selling price of the futures contract, it was only $160 for a $2,000 difference in the limit price.

The maximum gains for the options and stop order strategies were substantially larger than for the futures hedges as would be expected. These ranged from approximately $8,500 to $10,800 more than the maximum returns for the optimal hedge using only futures contracts. It was also the case that the maximum losses from both the options and stop order strategies were greater than for using only futures and also than for the initial series. The maximum losses for the stop orders occurred in each case when the mortgage price fell substantially but the futures position had been closed out.

The maximum losses for each of the options strategies were not representative for the series. For each call and put strategy, the maximum loss occurred in the period from April, 1980, to October, 1980, when the mortgage price rose by over ten points (for the reverse series, it fell by this amount). This was during the credit crunch of 1980 and the mortgage rate used for April was not reflective of actual rates, since the volume of lending declined substantially for this month. The futures prices for the initial and end-period dates were different by less than one point for this period. There are no other periods during the series when the loss for any option position was greater than ten points, and only two for which the loss was greater than seven points.

The performance of the stop orders vis-a-vis options strategies during a period of generally increasing prices depends on the limits established and the relationship between the option exercise prices and the initial futures prices. For example, the three point loss limit on the futures contract had a slightly lower mean return and larger standard deviation of return than did the call option strategy using in-the-money options. However, the five-point stop order strategy had a higher mean return than did the out-of-the-money call strategy and substantially the same standard deviation of returns. Thus, during the sample period, the performance of the stop order strategies vis-a-vis options strategies depended on the specific order limits and option exercise prices used.

Returns to the Alternative Strategies for the Combined Series

Combining the initial and reverse series would result in a sample period for which there was no drift in the prices of the mortgage and futures instruments, but substantial variation in the prices within the period. The mean returns for the unhedged mortgage position, and the hedged positions using only the T-bond futures contracts would be zero over the entire sample period. The standard deviations of the returns would simply be the values for either of the sample periods.

The mean returns and standard deviations of the returns for each of the alternative options and stop order strategies would be the average for the two time series. The mean and standard deviations of the returns for the alternative strategies are presented in Table 5-7.

Table 5-7. Mean and standard deviation of returns for alternative strategies: Combined series and lagged relationship (in thousands of dollars)

Strategies	Not corrected for autocorrelation		Corrected for autocorrelation	
	Mean return	Standard deviation of return	Mean return	Standard deviation of return
Unhedged mortgages	0	7.061	0	7.061
Mortgages and futures				
One-to-one hedge ratio	0	3.095	0	3.095
Risk-minimizing hedge ratio	0	2.896	0	2.975
Mortgages, futures and call options				
Exercise price > futures price	-.561	4.202	-.505	4.365
Exercise price = futures price	-.465	4.647	-.415	4.794
Exercise price < futures price	-.329	4.890	-.283	5.053
Mortgages and put options				
Exercise price > futures price	-.437	4.211	-.391	4.373
Exercise price = futures price	-.465	4.655	-.415	4.802
Exercise price < futures price	-.456	4.872	-.397	5.045
Mortgages, futures and stop orders				
Three point order	-.038	5.339	-.033	5.454
Four point order	.005	4.921	.001	5.091
Five point order	.043	4.517	.003	4.523

There is no clearly superior call options strategy for this series. Although buying call options with exercise prices less than the initial futures price would have resulted in the largest return, the standard deviation of the returns for this strategy was the highest of any of the call options strategies. This would not have been the case for the put options strategies. Purchasing puts with exercise prices higher than the initial futures prices would have provided the largest mean return and the smallest standard deviation of the returns.

Each of the alternative call and put options strategies had a lower mean return and a larger standard deviation than the futures/only strategies. Thus, ex post, the use of options would have been an inferior strategy to using only futures contracts in the hedge. The risk-return distributions obviously depend on the premiums paid for the options, but each of the options strategies had a mean return at least $283 less than the futures strategies. It is extremely unlikely that the estimated options prices could have been biased upward by this amount.

The mean returns for the stop order hedges were all extremely close to zero, but their standard deviations were substantially larger than was the case for the hedges using only futures contracts. Thus, the protection against upward movements in the futures price would have resulted in significantly greater variability in the returns. In retrospect, if stop orders were placed on the futures contracts, it would have been best to use stop orders placed five points above the initial futures price, since the standard deviation of the returns is smallest for this strategy.

As was true for both the initial and reverse series, it is not possible to conclude that using stop orders would have been superior to the use of futures options. Although the stop order strategies resulted in higher mean returns than did the options strategies, the standard deviations of the returns were larger for the three and four point stop orders than for any of the call or put options strategies.

Comparison of the Alternative Strategies

The risk-return distributions of the alternative hedging strategies estimated for the initial and reverse sample periods provide useful information for evaluating the performance of these strategies. For each of the strategies, the risk of the cash position, as measured by the standard deviation of the returns, was substantially reduced both in periods of declining and increasing prices (see Table A-7 of the Appendix for the efficiency in reducing the variance of returns for the alternative strategies). Each of the alternative strategies also produced a mean return closer to zero than did the unhedged position when prices increased or decreased. As expected, hedging reduces both the loss to the cash position caused by declining prices, and the gain to the position resulting from increasing prices.

The results of the simulations support the conclusion that using either options or stop orders increases the mean return for the hedge during a period of increasing cash and futures prices, but also increases the volatility of the return. The standard deviation of the returns was larger than when the hedged position contained only futures contracts for each of the alternative options or stop order strategies during both the initial and reverse series experiments.

The use of stop orders to limit losses on the futures position resulted in a substantial alteration in the risk-return distribution of the hedge. The closer the stop-out price to the initial futures price, the greater the volatility of the return during both periods of rising and falling prices. Also, the closer the stop loss limit to the initial futures price, the farther the mean return was from zero. There is, thus, no clearly superior choice of the level at which the stop order limit should be placed for both periods of increasing and decreasing cash prices. However, for a period of stationary prices containing substantial price movements within the period, using a higher stop order limit would result in a smaller variance of the returns without reducing the mean return.

As is the case with stop orders, there is no options strategy that provides the best risk-return distribution for both periods of increasing and decreasing prices. For both of the sample periods, the standard deviations of the returns were smallest when the exercise pric of the option was larger than the initial futures price. On the basis of risk reduction, this is clearly the best strategy. However, the mean return for the position using these exercise prices (although negative) was largest when mortgage prices were declining but smallest for periods of increasing prices. None of the exercise price levels provided both a higher mean return and smaller standard deviation of returns in both periods of declining and increasing prices.

As was discussed in Chapter III, placing stop orders on the futures position taken is an alternative to the purchase of options on the contracts. In each case, the purpose of the strategy is to protect against losses in periods when the hedged instrument's price is falling without foregoing the gains when the instrument's price increases. In selecting which of these alternatives is superior, it must be determined whether the premium paid for the option is offset by the probability that the stop order is exercised when, ex post, it should not have been.

For the initial price series, the loss for each of the stop order limits used was less than for any of the option strategies. This was not true for the reverse series comparisons. The options positions with exercise prices less than the initial futures prices had larger mean returns than for any of the stop order limits but, when the exercise prices were higher than the futures price, the mean returns were lower than for any of the stop orders.

The same results hold with respect to the risk distributions. For both the initial and reverse price series, the standard deviation of the returns of the stop order positions were in some cases less than, and in others greater than, those of the options positions. For both price series, the risk reduction potential of the futures/stop order hedge as compared to the futures/call option or put option hedge depends on the specific stop-out limits and options exercise prices. However, using five point stop orders did produce a larger mean return and a smaller standard deviation than when at-the-money options were used for both the initial and reverse sample periods.

Although the five point stop order outperforms the at-the-money option for both periods, there is no clearly superior strategy for any other pairwise comparisons. It is not possible to conclude that the stop order strategies are in general superior to options strategies, nor that the use of options provides a better risk-return than do stop order strategies.

CHAPTER VI. SUMMARY AND CONCLUSIONS

Review of the Empirical Results

The Black options pricing model together with the put-call parity equation provides good estimates of the prices of put and call options on Treasury bond futures contracts for the period for which these option contracts have been traded. For the sample period used in this study, these estimates exhibited no systematic bias for either the call or put options, and approximately forty percent of the estimates were within one-tenth of a point ($100) of the actual trading prices. There was also no evidence that the difference between the estimated and actual prices varied systematically with either the time to expiration of the option or the relationship between the futures contract price and the exercise price of the option. The improvement in the predictive power of the model for the 1983 period is best explained by the newness of the contract in 1982. As the market achieves greater maturity, the predictive power of the model can be retested using both contracts traded at a later date and a larger sample size.

As expected, the use of Treasury bond futures contracts to hedge mortgage price risk can substantially reduce the risk of the cash position. When the lagged relationship between futures and mortgage prices was incorporated in the hedging strategy, the variance of the returns was reduced by approximately seventy-three to eighty-two percent depending on the optimal hedge ratio used. Based on these results, it can be concluded that the risk reduction potential of hedging with futures contracts holds for both increases and decreases in mortgage prices.

The incorporation of either options on futures contracts or stop order limits on the futures contracts in the hedging strategy would substantially alter the risk-return distributions from that of the strategy which used only futures contracts. During periods of generally declining cash and futures prices, i.e., generally rising interest rates, any of the alternative options or stop order strategies tested would have resulted in a smaller mean return and a larger standard deviation of the returns than would the pure futures hedge. Assuming hedging is undertaken to reduce the risk of the position, each of the alternative options or stop order strategies have been inferior to hedges using only futures contracts. However, for the period of declining prices, all of the options and stop order strategies tested outperformed the unhedged position both in terms of providing a larger mean return and a smaller variance of the returns.

Each of the alternative options and stop order strategies provided a larger mean return than the pure futures hedge during the period of generally increasing prices, i.e., generally falling interest rates. However, each of these strategies also had a larger variance of the returns than the futures hedge. Thus, hedging using options or stop orders would reduce the mean return by less than would using only futures contracts during a period of increasing prices, but the variance of returns would be increased.

The results of the sample period mean-variance estimates presented in Table 5-2 through 5-5 suggest that there is no one options or stop order strategy that is clearly superior to all of the other strategies for both periods of declining and increasing cash instrument prices.

Selecting one option or one stop order strategy as optimal requires an evaluation of the hedger's tolerance for risk. In each case, the alternative positions have different risk-return distributions, depending on the direction of mortgage price changes.

Ex post, the estimated option prices used in the simulations appear to be reasonable. If the estimated prices were either too high or low, the options strategies would provide returns which were either lower or higher than the respective stop order strategies. This was not the case. For both periods of declining and increasing mortgage and futures prices, the mean returns for some of the stop order limits were larger than for some of the alternative options strategies. However, there was no one stop order strategy that provided a higher mean return than all of the options strategies for both sample periods. However, based on the results of the estimates for the combined sample period, it can be concluded that hedges using stop orders would provide a higher mean return than any of the alternative options strategies during periods when there is no general price trend for the cash and futures instruments. Additionally, the five point stop order provided a higher mean return and lower standard deviation than did the use of either call or put options with exercise prices equal to, or less than, the initial futures price. Thus, it would appear that this stop order would be better than either of these options strategies if the hedger had no expectations about the direction of prices' movements. Additionally, the generally larger variance of the returns for the stop order strategies as compared to the options strategy further limits the choice of any strategy as superior to the other strategies.

Limitations of the Study and Suggestions for Further Research

The empirical tests of alternative strategies to hedge against mortgage price risk which have been conducted in this study could be extended in a number of directions. Ideally, a daily price series for seasoned mortgages would be used to evaluate the hedging performance of the alternative strategies. However, there is currently no organized market which trades seasoned mortgages; consequently, daily trading data on mortgage prices do not exist.

The assumption that the mortgages being hedged are eight-percent twenty-five-year fixed-rate loans facilitates the estimation of the optimal hedge ratio and the estimation of the risk-return distributions of the alternative strategies. Extending the empirical tests to the hedging effectiveness of strategies for alternative fixed interest rates and other types of mortgage contracts would provide useful information for the mortgage holder. The recent growth in the use of alternative mortgage instruments, such as adjustable rate, graduated payment and balloon payment loans will undoubtedly alter the hedging strategies of institutions which hold mortgage portfolios. Reestimation of the optimal hedge ratios and estimation of risk-return distributions for these instruments is essential for effective hedging of interest rate risk.

As Merton, Scholes and Gladstein (1982, p. 3) have emphasized for simulations of stock options returns, the returns to the alternative futures options strategies are sensitive to the premiums paid for the options. Although there is no evidence of systematic bias in the simulated options prices used in this study, further testing of the Black model should be conducted as the futures options market matures. Further testing of the model would provide a check on the accuracy of the risk-return distributions estimated.

Although three different exercise prices for the options were used in this study, additional testing of options which are further in-the-money and out-of-the-money could be useful in evaluating how the mean-variance distribution of the returns to the cash position would be altered by using alternative option exercise prices.

The results of the stop order estimations support the conclusions that placing stop order limits further above the original futures prices will reduce the probability that the orders will be executed when, ex post, they should not have been. The results provide evidence that the greater the difference between the stop-out limit and the original futures price, the closer is the mean return to zero and the smaller the variance of the return.

A useful area of research is the determination of an optimal stop order limit. The larger (smaller) the difference between the stop-out price and the initial futures price, the smaller (greater) the probability that the order will be executed when, ex post, it should not have been. But increasing (decreasing) this difference will also reduce (increase) the return when the futures price increases over the hedging period. The difficulty in attempting to determine an optimal stop limit value is that one must know the probability that the futures price will surpass the stop limit during the period but fall below the initial futures price at the termination of the hedge. Thus far, there is little published research which evaluates the use of stop orders in hedging strategies, and none in which a theoretical analysis of the optimal stop order limit is developed.

As with any study based on past experience, one must exercise care in using the simulations undertaken in this study as an indicator of the future performance of the alternative strategies. The results are obviously dependent upon the trend of cash mortgage prices and the volatility of the prices. Future trends and volatility could be substantially different.

Finally, it should not be concluded that any strategy dominates any other one. If the options and the underlying futures contracts are correctly priced, there will be no best strategy for all hedgers. Since the patterns of returns of the various strategies are not the same, investors with different risk tolerances will have different optimal strategies.

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APPENDIX

Table A-1. Model parameters for estimating option prices

Date	Futures contract	Futures contract price	Time to contract expiration	Treasury bill rate	One-month historical variance
11/15/82	Mar. 1983	76.625	95	.0891	.03423
01/14/83	Mar. 1983	76.875	34	.0808	.01890
01/14/83	June 1983	76.156	126	.0816	.01890
01/26/83	June 1983	73.031	144	.0795	.01394
02/15/83	June 1983	73.219	94	.0876	.01874
03/15/83	June 1983	76.063	66	.0884	.02129
03/15/83	Sept. 1983	76.063	157	.0884	.02129
04/14/83	Sept. 1983	75.719	137	.0871	.01194
06/15/83	Dec. 1983	75.125	156	.0934	.01408

Table A-2. Actual and estimated call and put options prices

Date	Contract	Exercise price	Call options		Put options	
			Actual	Estimated	Actual	Estimated
10/26/82	December 1982	70	5.406	5.767	.156	.513
		72	3.625	4.247	.406	.983
		74	2.391	2.973	1.000	1.698
		76	1.234	1.969	1.859	2.685
		78	.594	1.232	3.156	3.937
12/27/82	June 1983	68	8.719	8.765	.563	.617
		70	7.094	7.212	1.000	1.003
		72	5.609	5.810	1.625	1.539
		74	4.469	4.577	2.469	2.245
		76	3.187	3.525	3.000	3.131
		78	2.313	2.651	4.469	4.196
		80	1.531	1.948	6.000	5.432
02/25/83	June 1983	68	9.000	8.928	.063	.060
		70	7.156	7.075	.156	.171
		72	5.406	5.439	.313	.410
		74	3.609	3.826	.594	.850
		76	2.313	2.568	1.266	1.555
		78	1.313	1.607	2.141	2.558
04/04/83	June 1983	68	8.250	8.164	.016	.003
		70	6.250	6.209	.047	.026
		72	4.313	4.335	.141	.131
		74	2.578	2.680	.391	.454
		76	1.234	1.413	1.016	1.116
04/04/83	September 1983	70	6.000	5.913	.406	.378
		72	4.375	4.369	.875	.770
		74	2.969	3.059	1.375	1.395
		76	2.047	2.019	2.297	2.292
		78	1.234	1.253	3.406	3.461

Table A-3. Returns to alternative hedging strategies: Contemporaneous mortgage and futures prices for initial series

Initial date	Change in mortgage prices	Change in futures price	One-to-one hedge ratio	Optimal hedge ratio	Stop orders: Three point	Stop orders: Four point
7/15/78	-3.86	-1.313	-2.547	-3.257	-5.526	-5.727
8/15/78	-3.55	-3.437	-0.113	-1.970	-1.970	-1.970
9/15/78	-3.77	-5.719	1.949	-1.142	-1.142	-1.142
10/15/78	-3.87	-3.905	0.036	-2.075	-2.075	-2.075
11/15/78	-3.68	-5.563	1.883	-1.123	-1.123	-1.123
12/15/78	-3.77	-0.906	-2.864	-3.354	-3.354	-3.354
1/15/79	-3.68	-0.281	-3.399	-3.551	-3.551	-3.551
2/15/79	-3.62	0.219	-3.839	-3.721	-3.721	-3.721
3/15/79	-4.02	-1.656	-2.365	-3.259	-3.259	-3.259
4/15/79	-4.98	-7.437	2.457	-1.562	-1.562	-1.562
5/15/79	-8.84	-8.437	-0.403	-4.962	-10.449	-4.962
6/15/79	-8.83	-9.954	1.124	-4.255	-4.255	-4.255
7/15/79	-8.92	-10.937	2.017	-3.893	-3.893	-3.893
8/15/79	-9.04	-22.469	13.429	1.287	1.287	1.287
9/15/79	-14.84	-19.531	4.691	-5.864	-5.864	-5.864
10/15/79	-20.03	-8.813	-11.217	-15.980	-15.980	-15.980
11/15/79	-12.14	-2.719	-9.421	-10.890	-13.763	-10.890
12/15/79	-1.74	3.406	-5.146	-3.305	-3.305	-3.305
1/15/80	2.40	-1.344	3.744	3.018	.748	-.228
2/15/80	2.87	-1.344	-3.849	-.218	1.348	.170
3/15/80	6.17	.608	5.482	5.854	3.958	3.958
4/15/80	10.01	-0.969	10.979	10.455	8.344	8.157
5/15/80	5.37	-10.750	16.120	10.311	3.977	3.359
6/15/80	-6.33	-18.406	12.076	2.129	2.129	2.129
7/15/80	-11.43	-9.937	-1.493	-6.863	-6.863	-6.863
8/15/80	-11.48	-10.719	-0.761	-5.554	-6.554	-6.554
9/15/80	-8.78	-3.812	-4.968	-7.028	-7.028	-7.028
10/15/80	-6.50	-10.781	4.281	-1.545	-1.545	-1.545
11/15/80	-6.74	-6.875	0.135	-3.580	-8.492	-8.636
12/15/80	-6.39	-0.406	-5.984	-6.203	-8.659	-8.659
1/15/81	-5.22	-7.094	1.874	-1.960	-1.960	-1.960
2/15/81	-6.47	-4.656	-1.814	-4.330	-8.064	-8.452
3/15/81	-7.38	-10.281	2.901	-2.655	-2.655	-2.655
4/15/81	-8.14	-6.156	-1.984	-5.311	-9.591	-9.993
5/15/81	-6.27	0.250	-6.520	-6.385	-7.807	-8.381
6/15/81	-2.04	-5.781	3.741	0.617	0.617	0.617
7/15/81	-1.54	-6.969	5.429	1.663	1.663	1.663
8/15/81	-0.56	-3.437	2.877	1.020	-2.082	-2.456
9/15/81	-0.80	2.219	-1.419	-0.220	-0.579	-1.268
10/15/81	2.34	3.687	-1.347	-.645	-.760	0.401
11/15/81	2.62	-0.469	3.089	2.836	2.836	2.836
12/15/81	1.10	-2.937	4.037	2.450	2.450	2.450
1/15/82	0.38	3.031	-2.651	-1.013	-1.430	-1.473

	Futures and call options			Put options		
Five point	E > F	E = F	E < F	E > F	E = F	E < F
-3.257	-3.394	-3.666	-4.031	-3.344	-3.658	-4.067
-1.970	-2.481	-2.826	-3.305	-2.440	-2.821	-3.335
-1.142	-1.686	-2.030	-2.498	-1.628	-2.022	2.540
-2.075	-2.450	-2.761	-3.217	-2.388	-2.747	-3.249
-1.123	-1.709	-2.060	-2.531	-1.660	-2.054	-2.569
-3.354	-3.722	-4.053	-4.123	-3.658	-4.047	-4.176
-3.551	-4.084	-4.250	-3.895	-4.053	-4.272	-3.970
-3.721	-4.265	-4.278	-3.908	-4.234	-4.292	-3.967
-3.259	-3.744	-4.120	-4.302	-3.695	-4.132	-4.375
-1.562	-1.719	-2.022	-2.569	-1.664	-2.035	-2.622
-4.962	-5.325	-5.713	-6.291	-5.292	-5.726	-6.349
-4.255	-5.337	-5.755	-6.280	-5.308	-5.784	-6.366
-3.893	-4.482	-4.889	-5.439	-4.450	-4.909	-5.510
1.287	0.713	0.296	-0.270	0.739	0.277	-0.334
-5.864	-6.423	-6.812	-7.342	-6.377	-6.832	-7.426
-15.980	-17.006	-17.393	-17.866	-16.955	-17.407	-17.947
-10.890	-12.673	-13.037	-13.037	-12.600	-13.021	-13.941
-3.305	-4.226	-3.694	-3.214	-4.175	-3.719	-3.316
-.228	2.252	1.926	1.512	2.349	1.957	1.478
.170	.306	.865	1.363	.378	.877	1.313
3.384	2.899	2.561	3.111	3.028	2.594	3.048
7.439	7.917	7.565	7.321	8.016	7.592	7.274
2.785	8.138	7.740	7.286	8.165	7.723	7.231
2.129	-0.295	-0.669	-1.082	-0.242	-0.656	-1.112
-6.863	-8.521	-8.893	-9.316	-8.467	-8.883	-9.351
-8.050	-8.435	-8.887	-8.010	-8.438	-8.932	-8.932
-7.028	-8.724	-9.112	-9.558	-8.677	-9.131	-9.645
-1.545	-3.700	-4.089	-4.526	-3.659	-4.110	-4.610
-9.268	-5.128	-5.477	-5.881	-5.040	-5.451	-5.918
-7.662	-8.503	-8.800	-8.286	-8.437	-8.828	-8.413
-1.960	-3.732	-4.075	-4.467	-3.618	-4.041	-4.513
-8.912	-5.399	-5.749	-6.176	-5.317	-5.736	-6.232
-2.655	-4.244	-4.597	-5.006	-4.150	-4.582	-5.068
-5.311	-7.160	-7.549	-7.995	-7.121	-7.588	-8.111
-6.385	-7.818	-7.878	-7.400	-7.757	-7.895	-7.496
0.617	-0.443	-0.775	-1.178	-0.332	-0.742	-1.232
1.663	0.507	0.165	-0.246	0.609	0.187	-0.303
1.020	-0.249	-0.629	-1.083	-0.194	-0.648	-1.175
-1.527	-1.399	-0.840	-0.344	-1.315	-0.850	-0.449
-0.188	-0.210	0.356	0.863	-0.123	0.364	0.794
2.836	1.003	0.737	1.204	1.035	0.717	1.131
2.450	0.135	-0.221	-0.618	0.216	-0.214	-0.686
-2.263	-2.314	-1.729	-1.203	-2.206	-1.695	-1.242

Table A-3. *Continued*

Initial date	Change in mortgage price	Change in futures price	One-to-one hedge ratio	Optimal hedge ratio	Stop orders: Three point	Stop orders: Four point
11/15/81	1.55	5.563	-4.013	-1.007	0.171	-0.360
12/15/81	4.33	4.719	-0.389	2.161	2.836	2.362
1/15/82	5.91	11.969	-6.059	0.409	4.402	3.555
2/15/82	9.30	12.312	-3.012	3.641	7.246	7.846
3/15/82	10.46	14.187	-3.727	3.940	8.837	8.246
4/15/82	11.97	14.281	-2.311	5.406	10.261	9.744
5/15/82	12.68	8.531	4.149	8.759	10.899	10.827
6/15/82	10.94	8.187	2.753	7.177	9.504	8.434
7/15/82	9.89	3.344	6.546	8.353	8.454	8.353
8/15/82	6.79	1.250	5.540	6.215	5.196	6.215
9/15/82	5.22	0.906	4.314	4.804	3.640	3.367
10/15/82	2.79	-3.981	6.771	4.620	1.354	4.620
11/15/82	-0.43	-1.531	1.101	0.274	-1.838	-2.297
12/15/82	-1.01	-4.656	3.646	1.129	-2.403	1.129
1/15/83	-4.00	-5.813	1.813	-1.328	-1.328	-1.328
2/15/83	-3.97	-5.500	1.530	-1.442	-1.442	-1.442
6/15/83	-3.84	-5.656	1.816	-1.240	-1.240	-1.240

	Futures and call options			Put options		
Five point	E > F	E = F	E < F	E > F	E = F	E < F
-0.748	-0.592	-0.036	-.448	-0.523	-0.036	0.379
1.515	1.826	2.414	2.937	1.946	2.450	2.890
3.555	3.983	4.536	5.008	4.052	4.531	4.930
6.586	7.944	8.462	8.866	7.981	8.438	8.781
7.731	8.382	8.964	9.467	8.485	8.985	9.405
8.896	10.478	11.010	11.439	10.527	10.990	11.349
10.382	10.524	11.093	11.579	10.582	11.106	11.548
8.484	8.283	8.835	9.331	8.348	8.839	9.274
8.353	6.462	7.022	7.536	6.526	7.042	7.512
6.215	4.777	5.103	5.560	4.812	5.097	5.513
4.801	3.211	3.544	3.999	3.245	3.527	3.930
4.620	3.661	3.301	2.856	3.713	3.310	2.821
-2.929	-0.712	-1.119	-1.095	-0.686	-1.133	-1.148
1.129	-0.2234	-0.634	-1.122	-0.195	-0.660	-1.202
-1.328	-1.9989	-2.336	-2.778	-1.937	-2.322	-2.810
-1.442	-2.2061	-2.607	-3.122	-2.178	-2.617	-3.173
-1.240	-2.2427	-2.642	-3.137	-2.205	-2.662	-3.213

Table A-4. Returns to alternative hedging strategies: Lagged mortgages and futures prices for initial series

Initial date	Change in mortgage price	Change in futures price	One-to-one hedge ratio	Optimal hedge ratio	Stop orders: Three point	Four point
8/15/78	-3.55	-1.313	-2.237	-2.5458	-6.322	-6.657
9/15/78	-3.77	-3.437	-0.333	-1.1414	-1.141	-1.141
10/15/78	-3.87	-5.719	1.649	0.5039	0.504	0.604
11/15/78	-3.68	-3.906	0.226	-0.6927	-0.693	-0.653
12/15/78	-3.77	-5.563	1.793	0.4846	0.485	0.485
1/15/79	-3.68	-0.906	-2.774	-2.9871	-2.987	-2.987
2/15/79	-3.62	-0.281	-3.339	-3.4051	-3.405	-3.405
3/15/79	-4.02	0.219	-4.239	-4.1875	-4.187	-4.187
4/15/79	-4.98	-1.656	-3.324	-3.7135	-3.713	-3.713
5/15/79	-8.84	-7.437	-1.403	-3.1522	-3.152	-3.152
6/15/79	-8.83	-8.437	-0.393	-2.3774	-11.507	-2.577
7/15/79	-8.92	-9.954	1.034	-1.3072	-1.307	-1.307
8/15/79	-9.04	-10.937	1.697	-0.6754	-0.675	-0.675
9/15/79	-14.84	-22.469	7.629	2.3443	2.344	2.344
10/15/79	-20.03	-19.531	-0.439	-5.0927	-5.093	-5.053
11/15/79	-12.14	-8.813	-3.327	-5.3998	-5.400	-5.400
12/15/79	-1.74	-2.719	0.979	0.3395	-4.441	0.339
1/15/80	2.40	3.406	-1.006	-0.2049	-0.205	-0.205
2/15/80	2.87	-1.344	4.214	3.8979	0.122	-1.504
3/15/80	6.17	6.719	-0.549	1.0313	6.637	1.677
4/15/80	10.01	0.688	9.322	9.4838	6.329	6.329
5/15/80	5.37	-0.969	6.339	6.1111	2.598	2.287
6/15/80	-6.33	-10.750	4.420	1.8916	-8.648	-0.676
7/15/80	-11.43	-18.406	6.976	2.6469	2.647	2.647
8/15/80	-11.48	-9.937	-1.543	-3.8802	-3.880	-3.880
9/15/80	-8.78	-10.719	1.939	-0.5821	-0.542	-0.582
10/15/80	-6.50	-3.812	-2.688	-3.5446	-3.515	-3.585
11/15/80	-6.74	-10.781	4.041	1.5053	1.805	1.505
12/15/80	-6.39	-6.875	0.485	-1.1320	-9.306	-9.545
1/15/81	-5.22	-0.406	-4.814	-4.9095	-8.996	-8.956
2/15/81	-6.47	-7.094	0.624	-1.0445	-1.045	-1.045
3/15/81	-7.38	-4.656	-2.724	-3.818	-10.033	-10.678
4/15/81	-8.14	-10.281	2.141	-0.2771	-0.277	-0.277
5/15/81	-6.27	-6.156	-0.114	-1.5619	-8.684	-9.353
6/15/81	-2.04	0.250	-2.290	-2.2312	-4.597	-5.553
7/15/81	-1.54	-5.781	4.241	2.3813	2.891	2.881
8/15/81	-0.56	-6.969	6.409	4.7699	4.770	4.770
9/15/81	0.80	-3.437	4.237	3.4256	-1.733	-2.355
10/15/81	2.34	2.219	0.121	0.6429	0.046	-1.102
11/15/81	2.62	3.687	-1.067	-0.1998	-0.009	-0.606
12/15/81	1.10	-0.469	1.563	1.4587	1.459	1.459
1/15/82	0.38	-2.937	3.317	2.6262	2.626	2.626

	Futures and call options			Put options		
Five point	E > F	E = F	E < F	E > F	E = F	E < F
-2.546	-2.774	-3.227	-3.835	-2.691	-3.213	-3.995
-1.141	-1.990	-2.566	-3.362	-1.922	-2.557	-3.412
0.504	-0.402	-0.974	-1.753	-0.306	-0.961	-1.823
-0.693	-1.316	-1.835	-2.592	-1.214	-1.810	-2.646
0.485	-0.490	-1.074	-1.859	-0.407	-1.064	-1.920
-2.987	-3.800	-4.150	-4.267	-3.493	-4.141	-4.356
-3.405	-4.292	-4.569	-3.978	-4.239	-4.605	-4.103
-4.187	-5.093	-5.114	-4.499	-5.042	-5.138	-4.597
-3.713	-4.520	-5.146	-5.448	-4.439	-5.167	-5.570
-3.152	-3.413	-3.917	-4.827	-3.322	-3.939	-4.916
-2.377	-2.981	-3.626	-4.587	-2.326	-3.648	-4.685
-1.307	-3.106	-3.803	-4.677	-3.059	-3.851	-4.819
-0.675	-1.655	-2.332	-3.247	-1.602	-2.564	-3.365
2.344	1.389	0.696	-0.245	1.433	0.664	-0.353
-5.093	-6.023	-6.671	-7.552	-5.947	-6.703	-7.693
-5.400	-7.108	-7.751	-8.537	-7.022	-7.775	-9.674
0.339	-2.626	-3.232	-3.232	-2.505	-3.206	-4.736
-0.205	-1.737	-0.851	-0.053	-1.651	-0.892	-0.223
-1.504	2.624	2.082	1.392	3.784	2.133	1.335
-1.623	1.802	2.833	3.662	2.022	2.853	3.583
5.373	4.566	4.004	4.920	4.781	4.058	4.815
1.092	1.887	1.302	0.875	2.052	1.345	0.917
-10.632	-1.724	-2.385	-3.141	-1.679	-2.414	-3.233
2.647	-1.392	-2.010	-2.696	-1.298	-1.988	-2.747
-3.880	-6.639	-7.257	-7.962	-6.549	-7.241	-8.020
-0.582	-3.071	-3.713	-4.46	-3.005	-3.718	-4.569
-3.585	-6.407	-7.051	-7.795	-6.328	-7.084	-7.939
1.505	-2.073	-2.727	-3.45	-2.012	-2.762	-3.594
-10.596	-3.707	-4.287	-4.960	-3.561	-4.245	-5.022
-9.283	-8.744	-9.230	-8.375	-8.626	-9.276	-8.586
-1.045	-3.994	-4.565	-5.216	-3.803	-4.508	-5.294
-11.443	-5.597	-6.190	-6.890	-5.460	-6.159	-6.984
-0.277	-2.921	-3.509	-4.189	-2.765	-3.483	-4.291
-1.562	-4.638	-5.287	-6.026	-4.574	-5.351	-6.222
-2.231	-4.616	-4.715	-3.920	-4.514	-4.744	-4.079
2.881	1.116	0.565	-0.108	1.316	0.619	-0.196
4.770	2.845	2.276	1.593	3.016	2.314	1.497
3.429	1.316	0.684	-0.071	1.408	0.653	-0.223
-1.532	-1.319	-0.388	0.436	-1.179	-0.405	0.262
-1.586	-1.623	-0.682	0.162	-1.479	-0.663	0.047
1.459	-1.590	-2.033	-1.256	-1.537	-2.066	-1.377
2.626	-1.226	-1.818	-2.479	-1.091	-1.806	-2.591

Table A-4. *Continued*

Initial date	Change in mortgage price	Change in futures price	One-to-one hedge ratio	Optimal hedge ratio	Stop orders: Three point	Four point
2/15/82	1.55	3.031	-1.481	-0.7681	-1.461	-1.533
3/15/82	4.33	5.563	-1.233	0.0754	2.036	1.151
4/15/82	5.91	4.719	1.191	2.3009	3.424	2.636
5/15/82	9.30	11.969	-2.669	0.1461	6.791	6.380
6/15/82	10.46	12.312	-1.852	1.0438	7.042	7.042
7/15/82	11.97	14.187	-2.217	1.1198	9.269	8.285
8/15/82	12.68	14.281	-1.801	1.7579	9.835	8.975
9/15/82	10.94	8.531	2.409	4.4155	7.976	7.857
10/15/82	9.89	8.187	1.703	3.6285	7.500	5.903
11/15/82	6.70	3.344	3.446	4.2326	4.400	4.233
12/15/82	5.22	1.250	3.970	4.2640	2.567	4.264
1/15/83	2.79	0.906	1.854	2.0971	0.161	-0.293
2/15/83	0.43	-3.981	3.551	2.6147	-2.820	2.615
3/15/83	-1.01	-1.531	0.521	0.1609	-3.352	-4.117
4/15/83	-4.00	-4.656	0.656	-0.43909	-6.3133	-0.439
5/15/83	-3.97	-5.813	1.943	0.47578	0.4758	0.475
6/15/83	-3.84	-6.500	1.660	0.36540	0.5664	0.366

	Futures and call options			Put options		
Five point	E > F	E = F	E < F	E > F	E = F	E < F
-2.848	-2.932	-1.959	-1.084	-2.753	-1.902	-1.149
0.506	0.765	1.690	2.495	0.880	1.690	2.380
1.226	1.742	2.721	3.592	1.942	2.781	3.514
5.380	6.092	7.013	7.799	6.207	7.005	7.668
5.943	8.203	9.065	9.738	8.265	9.025	9.596
7.429	8.811	9.481	10.317	8.694	9.515	10.213
7.565	10.196	11.083	11.796	10.278	11.049	11.646
7.116	7.351	8.299	9.108	7.448	8.321	9.056
5.803	5.464	6.387	7.212	5.577	6.393	7.116
4.233	1.086	2.017	2.873	1.192	2.050	2.833
4.264	1.870	2.412	3.173	1.928	2.402	3.094
2.097	-0.563	0.001	0.758	-0.497	-0.029	0.643
2.615	1.019	0.420	-0.320	1.105	0.435	-0.378
-5.169	-1.479	-2.157	-2.115	-1.435	-2.179	-2.205
-0.439	-2.691	-3.375	-4.187	-2.645	-3.418	-4.319
0.476	-0.640	-1.201	-1.937	-0.538	-1.177	-1.890
0.366	-0.908	-1.571	-2.430	-0.359	-1.599	-2.514

Table A-5. Returns to alternative hedging strategies: Contemporaneous mortgage and futures prices for reverse series

Initial date	Change in mortgage price	Change in futures price	One-to-one hedge ratio	Optimal hedge ratio	Stop orders: Three point	Stop orders: Four point
1/15/79	3.86	1.313	2.547	3.257	2.409	2.007
2/15/79	3.55	3.437	-.113	1.970	2.071	1.669
3/15/79	3.77	5.719	-1.949	1.142	2.291	1.798
4/15/79	3.87	3.906	-0.036	2.075	2.448	2.017
5/15/79	3.68	5.563	-1.883	1.123	2.229	1.784
6/15/79	3.77	0.906	2.864	3.354	3.354	3.354
7/15/79	3.68	0.281	3.399	3.551	3.551	3.551
8/15/79	3.62	-0.219	3.839	3.721	3.721	3.721
9/15/79	4.02	1.656	2.364	3.259	2.483	3.259
10/15/79	4.98	7.437	-2.457	1.562	2.926	2.926
11/15/79	8.84	8.437	0.403	4.962	7.231	6.915
12/15/79	8.83	9.954	-1.124	4.255	7.365	6.288
1/15/80	8.92	10.937	-2.017	3.893	7.398	6.723
2/15/80	9.04	22.469	-13.427	-1.287	7.575	6.557
3/15/80	14.84	16.531	-4.691	5.864	12.743	12.743
4/15/80	20.03	8.813	11.217	15.980	17.876	17.876
5/15/80	12.14	2.719	9.421	10.890	10.531	9.899
6/15/80	1.74	-3.406	5.146	3.305	3.305	3.305
7/15/80	-2.40	1.344	-3.744	-3.018	-4.282	-4.282
8/15/80	-2.87	-6.719	3.849	0.218	-4.292	-5.024
9/15/80	-6.17	-0.688	-5.482	-5.854	-7.578	-8.152
10/15/80	-10.01	-0.969	-10.979	-10.455	-11.805	-12.294
11/15/80	-5.37	10.750	-16.120	-10.311	-6.921	-7.567
12/15/80	6.33	18.406	-12.076	-2.120	4.937	4.219
1/15/81	11.43	9.937	1.493	6.863	9.735	9.304
2/15/81	11.48	10.719	0.761	6.554	9.713	9.627
3/15/81	8.78	3.812	4.968	7.028	7.358	6.697
4/15/81	6.50	10.781	-4.281	1.545	4.978	4.547
5/15/81	6.74	6.875	-0.135	3.580	5.332	4.485
6/15/81	6.39	0.406	5.984	6.203	4.896	4.408
7/15/81	5.22	7.094	-1.874	1.960	3.827	3.324
8/15/81	6.47	4.656	1.814	4.330	5.048	4.502
9/15/81	7.38	10.281	-2.901	2.655	5.558	5.498
10/15/81	8.14	6.156	1.984	5.311	6.618	6.258
11/15/81	6.27	-0.250	6.520	6.385	4.834	4.360
12/15/81	2.04	5.781	-3.741	-0.617	-.647	-0.071
1/15/82	1.54	6.969	-5.429	-1.663	-0.198	-0.672
2/15/82	0.56	3.437	-2.877	-1.020	-0.905	-1.379
3/15/82	-0.80	-2.219	1.419	0.220	-2.638	-2.782
4/15/82	-2.34	-3.687	1.347	-0.645	-3.848	-0,645
5/15/82	-2.62	0.469	-3.089	-2.836	-2.836	-2.836
6/15/82	-1.10	2.937	-4.037	-2.450	-2.551	-3.168

	Futures and call options			Put options		
Five point	E > F	E = F	E < F	E< F	E = F	E> F
1.375	2.764	3.136	3.534	2.808	3.128	3.473
1.037	2.404	2.961	3.335	2.443	2.956	3.285
1.314	2.651	3.261	3.647	2.713	3.262	3.587
2.075	2.393	3.009	3.465	2.463	3.023	3.425
1.368	2.193	2.738	3.160	2.236	2.734	3.110
3.354	2.773	3.191	3.548	2.805	3.165	3.464
3.551	3.019	3.002	3.384	3.057	2.988	3.318
3.721	3.230	3.101	3.438	3.256	3.082	3.373
3.259	2.252	2.771	3.209	2.294	2.746	3.118
2.639	2.276	2.908	3.291	2.325	2.790	3.205
6.384	6.640	7.180	7.651	6.696	7.177	7.589
6.288	6.717	7.296	7.795	6.791	7.320	7.740
6.378	5.981	6.532	7.033	6.053	6.538	6.974
6.728	5.384	5.932	6.443	5.443	5.921	6.372
12.542	11.195	11.780	12.328	11.338	11.826	12.276
17.876	16.922	17.483	17.998	17.010	17.497	17.938
9.842	5.317	9.835	10.304	9.343	9.819	10.246
3.305	0.659	0.275	-0.147	0.702	0.274	-0.195
-4.770	-4.727	-4.613	-4.125	-4.675	-4.607	-4.163
-5.197	-1.510	-1.901	-2.351	-1.474	-1.908	-2.401
-8.525	-7.998	-8.366	-8.177	-7.930	-8.366	-8.245
-12.638	-12.664	-12.454	-11.980	-12.611	-12.464	-12.032
-7.697	-8.503	-7.850	-7.442	-8.436	-7.948	-7.504
3.601	3.691	4.235	4.735	3.740	4.185	4.585
8.988	8.748	9.314	9.821	9.014	9.502	9.929
8.995	8.964	9.535	10.042	9.051	9.555	9.995
6.295	6.008	6.547	7.038	6.055	6.516	6.931
4.116	3.950	4.509	5.012	4.030	4.513	4.938
4.341	4.785	5.333	5.805	4.850	5.320	5.715
3.762	4.603	4.499	4.998	4.686	4.492	4.902
2.707	2.929	3.386	3.883	2.909	3.384	3.799
4.086	4.561	5.085	5.541	4.600	5.050	5.429
4.867	4.543	5.128	5.661	4.670	5.160	5.599
5.526	5.835	6.372	6.851	5.880	6.339	6.740
6.385	4.129	3.943	4.427	4.164	3.925	4.356
-0.660	-1.023	-0.454	0.068	-0.930	-0.434	0.014
-0.887	-0.736	-0.189	0.298	-0.674	-0.208	0.198
-1.795	-1.717	-1.124	-0.609	-1.618	-1.093	-0.645
0.220	-1.045	-1.418	-1.763	-0.980	-1.436	-1.864
-0.645	-1.398	-1.731	-2.157	-1.304	-1.711	-2.211
-2.836	-3.899	-3.932	-3.475	-3.846	-3.940	-3.545
-2.450	-2.592	-2.066	-1.634	-2.546	-2.102	-1.754

Table A-5. *Continued*

Initial date	Change in mortgage price	Change in futures price	One-to-one hedge ratio	Optimal hedge ratio	Stop orders: Three point	Four point
7/15/82	-0.38	03.031	2.651	1.013	-1.888	1.013
8/15/82	-1.55	-5.563	4.013	1.007	1.007	1.0070
9/15/82	-4.33	-4.719	0.389	-2.161	-2.161	-2.161
10/15/82	-5.91	-11.969	6.059	-0.409	-0.409	-0.409
11/15/82	-9.30	-12.312	3.012	-3.641	-3.641	-3.641
12/15/82	-10.46	-14.187	3.727	-3.940	-11.853	-3.940
1/15/83	-11.97	-14.281	2.311	-5.406	-5.406	-5.406
2/15/83	-12.68	-8.531	-4.149	-8.759	-14.131	-14.605
3/15/83	-10.94	-8.187	-2.753	-7.177	-7.177	-7.177
4/15/83	-9.89	-3.344	-6.546	-8.353	-8.353	-8.353
5/15/83	-6.79	-1.250	-5.540	-6.215	-6.215	-6.215
6/15/83	-5.22	-0.906	-4.314	-4.804	-6.800	-4.804
7/15/83	-2.79	3.981	-6.771	-4.620	-4.198	-4.743
8/15/83	0.43	1.531	01.101	-0.273	-1.121	-1.509
9/15/83	1.01	4.656	-3.646	-1.129	-0.383	-0.842
10/15/83	4.00	5.813	-1.813	1.328	2.592	2.061
11/15/83	3.97	5.500	-1.530	1.442	2.533	1.944
12/15/83	3.84	5.656	-1.816	1.240	2.432	1.771

	Futures and call options			Put optiosn		
Five point	E> F	E = F	E< F	E> F	E = F	E< F
1.013	-0.171	-0.538	-0.980	-0.105	-0.541	-1.052
1.007	-0.429	-0.793	-1.220	-3.375	-0.783	-1.255
-2.161	-4.367	-4.722	-5.131	-4.290	-4.717	-5.188
-0.409	-2.007	-2.391	-2.837	-1.965	-2.393	-2.884
-3.641	-5.256	-5.256	-5.675	-4.318	-5.237	-5.697
-3.940	-5.114	-5.478	-5.917	-5.054	-5.470	-5.960
-5.406	-6.639	-7.053	-7.551	-6.615	-7.072	-7.614
-8.759	-9.718	-10.092	-10.556	-9.677	-10.091	-10.596
-7.177	-8.018	-8.390	-8.860	-7.966	-8.391	-8.915
-8.353	-9.224	-9.616	-10.113	-0.187	-9.627	-10.169
-6.215	-6.989	-7.342	-7.651	-6.941	-7.334	-7.683
-4.804	-6.017	-6.376	-6.492	-5.949	-6.366	-6.537
-5.160	-5.041	-4.483	-3.896	-4.979	-4.473	-4.041
-1.868	-1.352	-1.382	-0.884	-1.288	-1.363	-0.910
-1.345	-0.621	-0.082	-0.356	-0.5723	-0.091	-0.289
-1.659	2.200	2.775	3.244	2.2599	2.785	3.205
1.543	2.244	2.872	3.363	2.3097	2.894	3.341
1.456	2.142	2.780	3.278	2.2306	2.808	3.247

Table A-6. Returns to alternative hedging strategies: Lagged mortgage and futures prices for reverse series

Initial date	Change in mortgage price	Change in futures price	One-to-one hedge ratio	Optimal hedge ratio	Stop orders: Three point	Stop orders: Four point
2/15/79	3.55	1.313	2.237	2.545	0.849	0.100
3/15/79	3.77	3.437	0.333	1.141	1.015	0.266
4/15/79	3.87	5.719	-1.849	-0.503	1.115	0.179
5/15/79	3.68	3.906	-0.226	0.692	1.032	0.230
6/15/79	3.77	5.563	-1.793	-0.484	1.069	0.239
7/15/79	3.68	0.906	2.774	2.987	2.905	2.905
8/15/79	3.62	0.281	3.339	3.405	3.379	3.379
9/15/79	4.02	-0.219	4.239	4.187	4.207	4.207
10/15/79	4.98	1.656	3.324	3.713	2.118	3.563
11/15/79	8.84	7.437	1.403	3.152	5.015	5.015
12/15/79	8.83	8.437	0.393	2.377	5.834	5.246
1/15/80	8.92	9.954	-1.034	1.307	6.192	4.136
2/15/80	9.04	10.937	-1.897	0.675	6.205	4.948
3/15/80	14.84	22.469	-7.629	2.344	12.112	10.962
4/15/80	20.03	19.531	0.499	5.092	16.125	16.125
5/15/80	12.14	8.813	3.327	5.399	8.128	8.128
6/15/80	1.74	2.719	-0.979	-0.339	-1.256	-2.433
7/15/80	-2.40	-3.406	1.006	0.204	0.515	0.515
8/15/80	-2.87	1.344	-4.214	-3.897	-6.374	-6.374
9/15/80	-6.17	-6.719	0.549	-1.031	-8.818	-10.182
10/15/80	-10.01	-0.688	-9.322	-9.483	-12.631	-13.701
11/15/80	-5.37	0.969	-6.339	-6.111	-8.713	-9.623
12/15/80	6.33	10.750	-4.420	-1.891	3.441	2.238
1/15/81	11.43	18.406	-6.976	-2.646	8.836	7.498
2/15/81	11.48	9.937	1.543	3.880	8.324	7.521
3/15/81	8.78	10.719	-1.939	0.582	5.490	5.330
4/15/81	6.50	3.812	2.688	3.584	3.852	2.622
5/15/81	6.74	10.781	-2.041	-1.505	3.905	3.102
6/15/81	6.39	6.875	-0.483	1.132	3.769	2.191
7/15/81	5.22	0.406	4.814	4.909	2.438	1.529
8/15/81	6.47	7.094	-0.624	1.044	3.876	2.939
9/15/81	7.38	4.656	2.724	3.819	4.732	3.716
10/15/81	8.14	10.281	-2.141	0.277	5.305	4.635
11/15/81	6.27	6.156	0.114	1.561	3.435	2.766
12/15/81	2.04	-0.250	2.290	2.231	-0.635	-1.517
1/15/82	1.54	5.781	-4.241	-2.881	-1.054	-2.392
2/15/82	0.56	6.969	-6.409	-4.769	-2.676	-3.559
3/15/82	-0.80	3.437	-4.237	-3.428	-3.528	-4.411
4/15/82	-2.34	-2.219	-0.121	-0.642	-5.764	-6.031

Five point	Futures and call options E > F	E = F	E < F	Put options E > F	E = F	E < F
-0.585	1.726	2.346	3.008	1.800	2.332	2.906
-0.412	1.863	2.790	3.412	1.927	2.781	3.329
-0.217	2.008	3.024	3.665	2.111	3.025	3.565
0.693	1.223	2.247	3.007	1.338	2.271	2.940
-0.078	1.295	2.202	2.905	1.366	2.196	2.822
2.905	2.021	2.716	3.311	2.074	2.673	3.171
3.379	2.521	2.492	3.127	2.584	2.469	3.018
4.207	3.370	3.156	3.718	3.415	3.124	3.609
3.713	2.039	2.902	3.631	2.108	2.861	3.479
5.015	4.339	5.226	6.029	4.421	5.196	5.887
4.743	5.170	6.068	6.852	5.261	6.062	6.747
4.690	5.405	6.368	7.198	5.527	6.407	7.107
4.810	4.149	5.066	5.900	4.269	5.076	5.801
10.962	8.757	9.668	10.518	8.837	9.649	10.400
16.125	13.965	14.938	15.849	14.203	15.014	15.763
7.742	6.968	7.902	8.759	7.115	7.925	8.659
-2.433	-2.956	-2.096	-1.315	-2.914	-2.122	-1.412
0.250	-4.199	-4.838	-5.543	-4.128	-4.840	-5.619
-6.813	-6.742	-6.553	-5.740	-6.657	-6.542	-5.904
-10.182	-3.907	-4.558	-5.307	-3.846	-4.569	-5.390
-13.930	-13.053	-13.665	-13.349	-12.939	-13.665	-13.463
-9.744	-9.787	-9.436	-9.615	-9.697	-9.453	-8.735
2.458	1.116	2.037	2.683	1.227	2.040	2.779
6.889	7.038	7.943	8.772	7.120	7.861	8.526
7.417	7.017	7.960	8.802	7.460	8.271	8.982
9.645	4.593	5.543	6.387	4.738	5.576	6.308
2.365	1.887	2.783	3.602	1.965	2.733	3.424
2.773	2.496	3.427	4.264	2.630	3.433	4.141
2.399	3.136	4.048	4.834	3.245	4.028	4.684
0.846	2.247	2.073	2.904	2.384	2.061	2.744
2.288	2.491	3.419	4.246	2.624	3.414	4.105
3.413	4.204	5.977	5.834	4.269	5.017	5.648
3.957	3.420	4.392	5.279	3.631	4.446	5.176
1.920	2.435	3.325	4.126	2.510	3.273	3.941
2.231	-1.523	-1.832	-1.026	-1.465	-1.863	-1.145
-2.953	-3.558	-2.610	-1.741	-3.401	-2.577	-1.331
-3.559	-3.228	-2.317	-1.506	-3.124	-2.348	-1.673
-4.720	-4.588	-3.602	-2.746	-4.424	-3.550	-2.805
-0.643	-2.747	-3.368	-3.642	-2.639	-3.399	-4.111

Table A-6. *Continued*

Initial date	Change in mortgage price	Change in futures price	One-to-one hedge ratio	Optimal hedge ratio	Stop orders: Three point	Stop orders: Four point
5/15/82	-2.62	-3.687	1.067	0.199	-5.428	0.536
6/15/82	-1.10	0.469	-1.569	-1.458	-1.501	-1.501
7/15/82	-0.38	2.937	-3.317	-2.626	-3.081	-4.232
8/15/82	-1.55	-3.031	1.418	0.768	-4.358	1.044
9/15/82	-4.33	-5.563	1.233	-0.075	0.431	0.431
10/15/82	-5.91	-4.719	-1.19	-2.300	-1.871	-1.871
11/15/82	-9.30	-11.969	2.669	-0.146	0.944	0.944
12/15/82	-10.46	-12.312	1.852	-1.043	0.078	0.078
1/15/83	-11.97	-14.187	2.217	-1.119	-14.564	0.173
2/15/83	-12.68	-14.281	1.601	-1.757	-0.457	-0.457
3/15/83	-10.94	-8.531	-2.409	-4.415	-13.641	-14.524
4/15/83	-9.89	-8.187	-1.703	-3.628	-2.883	-2.883
5/15/83	-6.79	-3.344	-3.446	-4.232	-3.928	-3.928
6/15/83	-5.22	-1.250	-3.970	-4.264	-4.150	-4.150
7/15/83	-2.79	-0.906	-1.884	-2.097	-5.732	-2.015
8/15/83	0.43	3.981	-3.551	-2.614	-2.191	-3.208
9/15/83	1.01	1.531	-0.521	-0.160	-1.878	-2.600
10/15/83	4.00	4.656	-0.656	0.439	1.405	0.549
11/15/83	3.97	5.813	-1.843	-0.475	1.348	0.359
12/15/83	3.84	5.500	-1.660	-0.366	-1.165	0.068

	Futures and call options			Put options		
Five point	E > F	E = F	E < F	E > F	E = F	E < F
0.200	-1.052	-1.606	-2.316	-0.896	-1.573	-2.406
-1.459	-3.228	-3.284	-2.523	-3.141	-3.297	-2.639
-2.626	-2.863	-1.988	-1.269	-2.786	-2.048	-1.467
0.768	-1.202	-1.813	-2.548	-1.092	-1.818	-2.669
-0.075	-2.465	-3.070	-3.780	-2.375	-3.054	-3.838
-2.301	-5.954	-6.562	-7.244	-5.844	-6.554	-7.338
-0.146	-2.806	-3.445	-4.187	-2.736	-3.448	-4.264
-1.044	-3.730	-3.730	-4.427	-2.169	-3.619	-4.465
-1.120	-3.074	-3.680	-4.410	-2.974	-3.667	-4.483
-1.758	-3.510	-4.498	-5.327	-3.769	-4.530	-5.431
-4.415	-6.012	-8.534	-7.406	-5.943	-6.632	-7.471
-3.629	-6.028	-5.646	-6.428	-4.942	-5.643	-6.520
-3.928	-6.681	-6.335	-7.160	-5.621	-6.352	-7.255
-4.150	-5.551	-6.138	-6.654	-5.471	-6.125	-6.785
-2.015	-4.116	-4.714	-4.907	-4.003	-4.637	-4.982
-3.513	-3.316	-2.387	-1.580	-3.212	-2.370	-1.651
-2.814	-1.955	-2.006	-1.176	-1.849	-1.974	-1.220
0.080	1.285	2.182	2.912	1.366	2.167	2.801
0.074	0.974	1.931	2.712	1.074	1.949	2.648
-0.199	-0.968	2.014	2.831	1.077	2.049	2.793

Table A-7. Percentage reduction in risk for alternative hedging strategies: Corrected hedge ratio

Strategy	Initial series		Reverse series	
	Contemporaneous	Lagged	Contemporaneous	Lagged
Mortgages and futures				
One-to-one hedge ratio	.4648	.8079	.4648	.8079
Optimal hedge ratio	.4965	.8225	.4965	.8225
Mortgages, futures and call options				
Exercise price > futures price	.3846	.6836	.3424	.5457
Exercise price = futures price	.3359	.6164	.2833	.4545
Exercise price < futures price	.2869	.5423	.2244	.4302
Mortgages & put options				
Exercise price > futures price	.3837	.6819	.3172	.5448
Exercise price = futures price	.3350	.6150	.2821	.4529
Exercise price < futures price	.2787	.5390	.2249	.4377
Mortgages, futures and stop orders				
Three point orders	.3406	.5227	.2135	.2707
Four point order	.3919	.5875	.2908	.3602
Five point order	.4560	.6410	.3629	.5347